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The pion in electromagnetic and weak neutral current nuclear response functions*

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Submitted to: *Nuclear Physics A*

CTP#2194

March 1993

* This work is supported in part by funds provided by the U.S. Department of Energy (D.O.E.) under contract #DE-AC02-76ER03069.

Abstract

The impact of pionic correlations and meson–exchange currents in determining the (vector) response functions for electroweak quasielastic lepton scattering from nuclei is discussed. The approach taken builds on previous work where the Fermi gas model is used to maintain consistency in treating forces and currents (gauge invariance) and to provide a Lorentz covariant framework. Results obtained in first–order perturbation theory are compared with infinite–order summation schemes (HF and RPA) and found to provide quite successful approximations for the quasielastic response functions. The role of pionic correlations in hardening the responses R_L and R_T is investigated in some detail, including studies of the relative importance of central and tensor pieces of the force and of exchange and self–energy diagrams; in addition, their role in significantly modifying the longitudinal parity–violating response R_{AV}^L is explored. The MEC are shown to provide a small, but non–negligible, contribution in determining the vector responses.

1. Introduction

The role of the pion in nuclear structure has been explored for some time now ¹⁾ and clear signatures of its presence as a carrier of the force acting between nucleons via the one-pion-exchange potential (OPEP) have been found, for example, in studies of NN-scattering ²⁾ or of the deuteron (see ref. ³⁾ for a review in this area) and as a carrier of an electromagnetic current in np radiative capture ⁴⁾ or in the electrodisintegration of the deuteron ^{5,6)}. The properties of the 3-body nuclei ³He and ³H also appear to be represented quite well by hadronic descriptions which include pionic degrees of freedom in forces and currents, except perhaps at the highest momentum transfers in electron scattering where extremely short distance scales are being probed. Importantly, this success requires that the theory take into account 2-body meson-exchange currents (MEC) and 3-body forces (see, *e.g.*, refs. ^{7,8)}), both having parts that arise from pion exchange. Such observations regarding 2- and 3-body nuclei support the description of nuclei in general within the framework of a hadronic quantum field theory (the nucleus viewed as a composite system of baryons — especially nucleons — and mesons — especially pions).

However, when one comes to consider heavier nuclei the situation then becomes less clear because of the complexity of the nuclear many-body problem and the need to deal with less developed nuclear wave functions than are available for the few-body nuclei. To mention just a few examples where pionic effects have been sought, on the one hand the enhancement of the dipole photoabsorption sum rule over the canonical Thomas-Reiche-Kuhn value has been interpreted with some success in terms of a pionic contribution stemming mostly from the tensor force in second order ⁹⁾. On the other hand, evidence for the long-sought-after softening of the quasielastic peak in the spin-longitudinal $\sigma \cdot \mathbf{q}$ isovector nuclear response, a precursor of pion condensation at nuclear density, has never been convincingly found ^{10,11)}. Furthermore, while MEC effects in electron scattering from $A > 3$ nuclei are expected (see, *e.g.*, ref. ¹²⁾), they have been difficult to isolate from uncertainties in nuclear structure.

In contemporary descriptions of many-body nuclear structure using hadronic

degrees of freedom different starting points have been assumed. For example, in the relativistic mean-field approach of Walecka and collaborators ¹³⁾ (so-called relativistic quantum hadrodynamics, QHD) in lowest-order the pionic degrees of freedom do not enter; in that case, symmetry considerations (translational invariance) prevent the pion from contributing to the energy of the system. On the other hand, variational approaches (see refs. ^{7,8)}) suggest an important role for the pion: for example, in the work reported in ref. ¹⁴⁾ it is found that over half of the nuclear matter binding energy may be ascribed to the pion.

As the above issues are not yet fully resolved, we are left with some uncertainty about the size of the role played by pionic degrees of freedom in many-body nuclear structure. When attempting to elucidate this issue, it should be kept in mind that these degrees of freedom may be of rather different importance in describing different nuclear observables. In particular, their role in accounting for the binding energy of nuclei may be rather different from that involved in describing the electroweak quasielastic nuclear response functions which are the main focus of the present work. In the former, (particle-particle) interactions between nucleons in the Fermi sea are presumed to provide most of the binding energy, whereas in the latter particle-hole excitations involving particles typically at several hundred MeV in the continuum are dominant.

In this paper, which enlarges on a previous one ¹⁵⁾, we do not aim at establishing a relativistic quantum field theory of nuclei based on hadronic degrees of freedom or to explore ground-state properties using sophisticated nuclear wave functions. Rather we treat pionic degrees of freedom as perturbations, building on a simple, tractable, covariant nuclear model for quasielastic response functions, namely, the relativistic Fermi gas (RFG) model ¹⁶⁾. The free RFG model appears to describe nuclear excitations at high three-momentum transfer q and high energy-transfer ω reasonably well for kinematics near the quasielastic peak (QEP), that is, where $\omega \approx |Q^2|/2m_N$, with $Q^2 = \omega^2 - q^2$. To the extent that pionic correlations and pionic MEC provide only moderate corrections to the free RFG responses, one may hope that a perturbative treatment is meaningful. Accordingly, we start with all

Feynman diagrams carrying one pionic line and later extend our analysis to account for perturbative diagrams up to infinite order to test their importance. Issues relating to Lorentz covariance and electroweak gauge invariance can be explored in this model where a high level of consistency in treating forces and currents can be maintained. Some of the issues relating to gauge invariance in this pionic model are addressed in the present work. In subsequent work it will be important to explore the roles played by mesons heavier than the pion within the same framework, however, the present scope has been restricted to a study of the pionic effects in quasielastic electroweak nuclear response functions. Given the long-ranged character of pion exchange we believe that such effects provide a natural starting point for a more ambitious study.

In the present work we study the pionic correlation effects embodied in the so-called self-energy and exchange diagrams, whereas, for the MEC contributions we shall mostly rely on past work ¹⁵⁾, paying special attention to the problem of fulfilling the continuity equation in achieving a consistent treatment of currents and forces. In accord with those previous studies, we find sizable pionic contributions to the electromagnetic longitudinal (spin scalar, $\sigma = 0$) and transverse (spin vector, $\sigma = 1$) nuclear responses. In both cases it is found that the correlation effects produce a hardening of the responses, that is, a shift of the strength to higher ω .

In particular, as discussed in detail in the following sections, from our past and present analyses the following points emerge:

- a) In isospace the contribution of the self-energy diagram to the charge response is almost equally split between isoscalar ($\tau = 0$) and isovector ($\tau = 1$) components. (The latter, on the other hand, is of course overwhelming in the transverse response, due to the dominance of the isovector magnetic moment.) In contrast, the $\tau = 0$ part of the exchange diagram is three times as large as the $\tau = 1$ one in the charge response and this imbalance, which becomes even stronger in higher orders of perturbation theory, is further strengthened by the difference between the isoscalar and isovector form factors (see later). The isoscalar dominance of the pionic exchange correlations has dramatic con-

sequences for the weak neutral current longitudinal response function, as we shall see in sect. 4.

- b) While the tensor component of the OPEP never contributes to the self-energy in a translationally invariant system, the exchange diagram gets a tensor contribution, *but only in the transverse channel* and mostly via the backward-going graphs. This implies a different role for the pionic force in the two electromagnetic responses, a finding which should be tested against experiment.
- c) When we extend our analysis from first to infinite order of perturbation theory, thus generating the Hartree-Fock (HF) approximation with the self-energy diagram and the random phase approximation (RPA) with the exchange diagram, our results do not change substantially. It therefore appears that, although the pionic interaction is strong, nevertheless for quasielastic kinematics its effects are reasonably small at high- q , thus rendering perturbation theory quite accurate already at the lowest order, at least for the classes of diagrams studied here.
- d) Finally, the contribution of the central component of the pionic interaction to the exchange diagram stems largely from the δ -force and not from the finite-range one. In keeping with the usual approach taken in studies of pionic effects, we include a π NN vertex function Γ_π , whose scale is set by a mass parameter Λ_π , to smear out the δ -piece of OPEP. Consequently, one of the goals in the present work is to explore the sensitivity of the resulting nuclear quasielastic responses to the choice of the phenomenological parameter Λ_π . The question then arises: To what extent is the continuity equation modified by the presence of Γ_π ? We address this issue later in the present work and here simply recall that for pointlike nucleons the continuity equation is indeed obeyed by the OPEP and by the related pionic MEC ¹²⁾, but is not affected by the δ -interaction. This conclusion must be modified, however, when the vertex function Γ_π is introduced, although, as discussed below, additional MEC can be introduced ¹⁷⁾ in a way such that the continuity equation is still satisfied.

In contrast to the exchange diagram, the self-energy term gets a contribution only from momentum-dependent forces, and therefore an unmodified pionic δ -interaction does not contribute in this channel; in fact, the contribution coming from the δ -function in OPEP, when modified by the π NN vertex form factor, contains an effective momentum-dependence and so is nonzero.

Regarding the two parameters that characterize our approach, namely the Fermi momentum k_F and the mass parameter Λ_π , we note that the value of the first is essentially set by the nuclear density. Ultimately, given extensive evaluation of all nuclear model dependences (correlation effects, MEC effects, *etc.*), electron scattering measurements in the quasielastic region should serve to determine k_F . Likewise, some information on the range of acceptable values for the phenomenological parameter Λ_π can in principle be obtained from comparison with experiment. Clearly the value deduced for this parameter is model-dependent: it is meant to absorb effects from hadronic physics other than the explicit pionic degrees of freedom (to the extent that this is even possible through a single vertex function Γ_π). In an extended model with other active mesonic degrees of freedom presumably the physics embodied in this way can be quite different. Even given the limited scope of the present pionic model, it is interesting to explore the sensitivity in our results to the actual choice of Λ_π and we do so in sect. 5. The unexpected (and perhaps important) finding in this connection is the following: the diagrams which contribute most to the nuclear responses are those that are least affected by Γ_π .

With respect to the relativistic aspect of the present approach, one limitation of our treatment arises from the static character of our pionic interaction, while another arises from the not-fully-relativistic character of the fermion propagators that we use. However we have been able to achieve an almost exact relativistic treatment of the fermion kinematics. As discussed in ref. ¹⁵⁾, this is obtained by expressing the electromagnetic longitudinal and transverse responses and, as well, their weak neutral current analogs in terms of a single relativistic *scaling* variable ¹⁶⁾. Moreover we use an approximation to the relativistic electromagnetic and weak neutral current vertices, which has proven to be quite accurate. As we

shall see, this result is of relevance because the pionic effects in these observables are felt up to quite large momentum transfers (~ 1 GeV/c) where relativity cannot be ignored.

The items elaborated upon in the present paper are treated in the following order: in sect. 2 we deal with the self-energy diagram both in first-order perturbation theory and in the HF scheme. In sect. 3 we consider the exchange diagram, again both in first- and infinite-order perturbation theory, where the latter is explored within the framework of continued fractions. In first-order, both forward- and backward-amplitudes are included in the analysis. In sect. 4 we calculate the electromagnetic and the weak neutral current responses, focusing in particular on the large enhancement found for the weak neutral current longitudinal response when correlations are included, compared with the results found with the free RFG model where a delicate cancellation occurs. In the same section we also address the question of the evolution with q of both the correlation and MEC contributions. In sect. 5 we study the Λ_π - and k_F -dependences of our results. In the concluding section we summarize our results as they stand at present in treating pionic effects and their impact on quasielastic nuclear response functions.

2. Self-Energy Contributions

In this section we explore the contribution of the self-energy diagrams to the longitudinal and transverse electromagnetic responses of an infinite, homogeneous nuclear system with an equal number of protons and neutrons ($Z = N = A/2$). The corresponding (vector) weak neutral current responses are considered in sect. 4, whereas the axial-vector weak neutral current response is studied in a companion paper¹⁸). We shall confine ourselves to a consideration of the one-particle-one-hole (1p-1h) sector of the excitation spectrum of the RFG and, as already stated in sect. 1, in our scheme only the pion mediates the interaction between the nucleons.

The nuclear responses may be expressed via the polarization propagator. In first-order perturbation theory the pion yields two self-energy contributions to the latter, by dressing either the particle or the hole propagation in the associated Goldstone diagrams. These are displayed in fig. 1 and the corresponding analytic expressions for both the longitudinal and transverse channels have been derived in ref.¹⁵):

$$\begin{aligned} \Delta R_{L,T(s.e.)}^{corr}(q, \omega) &= \mathcal{C}_{L,T} \frac{1}{m_N^4} \lim_{\alpha \rightarrow 0} \frac{\partial}{\partial \alpha} \\ &\times \int \frac{d\mathbf{k}}{(2\pi)^3} \theta(k_F - k) \theta(|\mathbf{q} + \mathbf{k}| - k_F) \delta\left(\omega + \alpha - \frac{|Q^2|}{2m_N} - \frac{\mathbf{q} \cdot \mathbf{k}}{m_N}\right) \\ &\times \int \frac{d\mathbf{p}}{(2\pi)^3} \theta(k_F - p) \left\{ \Gamma_\pi^2(\mathbf{p} - \mathbf{k}) \frac{(\mathbf{p} - \mathbf{k})^2}{(\mathbf{p} - \mathbf{k})^2 + m_\pi^2} - \Gamma_\pi^2(\mathbf{q} + \mathbf{k} - \mathbf{p}) \frac{(\mathbf{q} + \mathbf{k} - \mathbf{p})^2}{(\mathbf{q} + \mathbf{k} - \mathbf{p})^2 + m_\pi^2} \right\} \end{aligned} \quad (2.1a)$$

$$\begin{aligned} &= \mathcal{C}_{L,T} \frac{1}{8\pi^2 \kappa} \left\{ \theta(\lambda_2 - \hat{\lambda}) \theta(\hat{\lambda} - |\lambda_1|) \right. \\ &\times \frac{1}{2\kappa} \left[\left(\eta_F \bar{\psi}_r \right) \mathcal{G} \left(\eta_F \bar{\psi}_r \right) - \left(\eta_F \psi_r \right) \mathcal{G} \left(\eta_F \psi_r \right) \right] \\ &\left. - \theta(\lambda_2 - \hat{\lambda}) \theta(\hat{\lambda} - \lambda_1) \mathcal{G} \left(\sqrt{\eta_F^2 + 4\hat{\lambda}} \right) + \theta(-\lambda_1 - \hat{\lambda}) \mathcal{G} \left(\sqrt{\eta_F^2 - 4\hat{\lambda}} \right) \right\}, \end{aligned} \quad (2.1b)$$

where $\xi_A = 3\pi^2 A$, $\lambda_2 = \kappa^2 + \eta_F \kappa$, $\lambda_1 = \kappa^2 - \eta_F \kappa$, $\hat{\lambda} = \lambda(\lambda + 1)$, $\bar{\psi}_r = \psi_r + 2\kappa/\eta_F$,

$$\mathcal{C}_L = f_L^2(Q^2) \frac{6\xi_A m_N^4}{k_F^3} \frac{f_\pi^2}{m_\pi^2} = f_L^2(Q^2) \frac{6\xi_A m_N}{\eta_F^3} \frac{f_\pi^2}{m_\pi^2}, \quad (2.2a)$$

$$C_T = f_T^2(Q^2) \frac{6\xi_A m_N^4}{k_F^3} \frac{q^2}{4m_N^2} (\mu_V^2 + \mu_S^2) \frac{f_\pi^2}{m_\pi^2} = f_T^2(Q^2) \frac{6\xi_A m_N}{\eta_F^3} \kappa^2 (\mu_V^2 + \mu_S^2) \frac{f_\pi^2}{m_\pi^2} \quad (2.2b)$$

and

$$\begin{aligned} \mathcal{G}(\beta) = & \frac{1}{4\pi^2} \left\{ \mu_\pi^3 \left[\tan^{-1}\left(\frac{\eta_F + \beta}{\mu_\pi}\right) + \tan^{-1}\left(\frac{\eta_F - \beta}{\mu_\pi}\right) \right] \right. \\ & - \frac{\mu_\pi^2}{4\beta} (\eta_F^2 + \mu_\pi^2 - \beta^2) \ln \left| \frac{(\eta_F + \beta)^2 + \mu_\pi^2}{(\eta_F - \beta)^2 + \mu_\pi^2} \right| \\ & + \frac{\lambda_\pi}{2} (\lambda_\pi^2 - 3\mu_\pi^2) \left[\tan^{-1}\left(\frac{\eta_F + \beta}{\lambda_\pi}\right) + \tan^{-1}\left(\frac{\eta_F - \beta}{\lambda_\pi}\right) \right] \\ & \left. + \frac{1}{4\beta} [\mu_\pi^2 (\eta_F^2 - \beta^2) - \lambda_\pi^2 (\lambda_\pi^2 - 2\mu_\pi^2)] \ln \left| \frac{(\eta_F + \beta)^2 + \lambda_\pi^2}{(\eta_F - \beta)^2 + \lambda_\pi^2} \right| \right\}. \end{aligned} \quad (2.3)$$

Here the contribution associated with the convective nucleon current has been neglected, since it represents only a tiny fraction (a few percent, at most) of the total transverse response for momentum transfers exceeding about 300 MeV/c. Note that in (2.1) the derivative and the limit with respect to the variable α define the prescription one should follow in order to deal with the double pole characterizing the first-order self-energy contribution to the nuclear responses.

In the above, along with the Fermi momentum k_F , the three-momentum transfer \mathbf{q} , the energy transfer ω , and the spacelike four-momentum transfer $Q^2 = \omega^2 - q^2 < 0$, with $q = |\mathbf{q}|$, the dimensionless variables of ref. ¹⁵⁾ have also been employed:

$$\kappa = \frac{q}{2m_N}, \quad \lambda = \frac{\omega}{2m_N}, \quad \eta_F = \frac{k_F}{m_N}, \quad \tau = \frac{|Q^2|}{4m_N^2}, \quad \mu_\pi = \frac{m_\pi}{m_N} \quad \text{and} \quad \lambda_\pi = \frac{\Lambda_\pi}{m_N}. \quad (2.4)$$

Moreover, $\mu_V = \mu_p - \mu_n$ and $\mu_S = \mu_p + \mu_n$ are the isovector and the isoscalar magnetic moments of the nucleon, respectively. To incorporate some aspects of the π NN vertex, we employ the monopole form

$$\Gamma_\pi(p) = \frac{\Lambda_\pi^2 - m_\pi^2}{\Lambda_\pi^2 + p^2}, \quad (2.5)$$

which will be discussed at some length later on, especially in connection with the problem of choosing an appropriate value for the phenomenological constant Λ_π .

Notice that in the energy-conserving δ -function occurring in our expressions for the self-energy contributions to R_L and R_T we have replaced the non-relativistic term $q^2/2m_N$ with the relativistic one $|Q^2|/2m_N$. This can be shown¹⁵⁾ to be almost exactly equivalent to employing relativistic kinematics and leads to the scaling variable of the relativistic Fermi gas model¹⁶⁾ which enters in (2.1):

$$\psi_r = \frac{1}{\eta_F} \left[\frac{\lambda(\lambda+1)}{\kappa} - \kappa \right] \quad (2.6a)$$

(the last terms on the right-hand side of (2.1b) correspond to the Pauli-blocked region, where no scaling occurs).

Had we kept the term $q^2/2m_N$ in the energy-conserving δ -function, then we would have obtained the same results but with the non-relativistic scaling variable

$$\psi_{nr} = \frac{1}{\eta_F} \left(\frac{\lambda}{\kappa} - \kappa \right) \quad (2.6b)$$

rather than ψ_r . Actually, an additional factor $1+2\lambda$ also appears in the “relativized” responses: it expresses the Jacobian of the transformation from the variable ψ_{nr} to ψ_r and has been included in the multiplicative form factors (see below).

Another place where relativity is carefully accounted for in our approach is in the electromagnetic vertices. Indeed, the electromagnetic form factors $f_{L,T}(Q^2)$ presently employed are given by the following expressions:

$$f_L^2(Q^2) = (1+2\lambda) \left\{ \frac{1}{1+\tau} [G_E^{m_t}(\tau)]^2 + \tau [G_M^{m_t}(\tau)]^2 \frac{\eta_F^2}{2} (1-\psi_r^2) \right\} \quad (2.7a)$$

$$f_T^2(Q^2) = (1+2\lambda) \frac{2\tau}{(\varepsilon+\lambda)^2} [G_M^{m_t}(\tau)]^2, \quad (2.7b)$$

with

$$G_E^{m_t}(\tau) \equiv \left(\frac{1}{2} + m_t\right) G_{E_p}(\tau) + \left(\frac{1}{2} - m_t\right) G_{E_n}(\tau) \quad (2.8a)$$

$$G_M^{m_t}(\tau) \equiv \left(\frac{1}{2} + m_t\right) G_{M_p}(\tau) + \left(\frac{1}{2} - m_t\right) G_{M_n}(\tau), \quad (2.8b)$$

$G_{E_{p,n}}$ and $G_{M_{p,n}}$ being the Sachs form factors of the proton and neutron and m_t labeling the isospin projection. The results in (2.1) are actually obtained by

adding the proton response ($m_t = 1/2$) multiplied by Z to the neutron response ($m_t = -1/2$) multiplied by N . Formulae (2.7) and (2.8) are deduced in ref. ¹⁵⁾ and there shown to provide an accurate representation of the RFG responses. We recall that the need for a fully-relativistic treatment of the γNN vertex arises from the existence in the problem of the non-relativistic reduction of the nuclear responses of a second scale involving κ and the squares of the nucleon magnetic moments ¹⁶⁾, beyond the usual one set by κ alone.

A final comment on the expressions (2.1b), already discussed in ref. ¹⁵⁾, relates to the domain in the (q, ω) -plane where $\Delta R_{L(s.e.)}^{corr}$ and $\Delta R_{T(s.e.)}^{corr}$ are nonzero. It is the same as that of the RFG, although the self-energy contributions do not vanish on the boundaries of the response region and are discontinuous across the boundary dividing the Pauli-blocked region from the non-blocked one (see the case at $q = 300$ MeV/c below in fig. 2). This is connected with the fact that the derivative of the RFG response does not vanish on the boundaries themselves and is discontinuous as well when one crosses into the Pauli-blocked region.

As already mentioned, in the present approach we treat the pionic lines of fig. 1 at the static level and hence they correspond to the well-known one-pion-exchange potential (OPEP):

$$\begin{aligned} V(q) &= -\frac{f_\pi^2}{m_\pi^2} \boldsymbol{\tau}_1 \cdot \boldsymbol{\tau}_2 \boldsymbol{\sigma}_1 \cdot \mathbf{q} \boldsymbol{\sigma}_2 \cdot \mathbf{q} \frac{1}{q^2 + m_\pi^2} \\ &= -\frac{1}{3} \frac{f_\pi^2}{m_\pi^2} \boldsymbol{\tau}_1 \cdot \boldsymbol{\tau}_2 [\boldsymbol{\sigma}_1 \cdot \boldsymbol{\sigma}_2 + S_{12}(\hat{\mathbf{q}})] \left(1 - \frac{m_\pi^2}{q^2 + m_\pi^2} \right). \end{aligned} \quad (2.9)$$

In (2.9) one easily recognizes the central contact and momentum-dependent pieces of the interaction in addition to the tensor contribution. By performing spin traces (see Appendix A), it is then an easy matter to show that *the tensor force S_{12} never contributes to the self-energy diagrams of fig. 1, i.e., in either longitudinal or transverse channels.*

It is equally easy to show, by the same token, that the longitudinal isoscalar ($\tau = 0$) and the isovector ($\tau = 1$) self-energy contributions are identical, but for the difference between the isoscalar and the isovector form factors $f_L^{\tau=0}$ and

$f_L^{\tau=1}$. This difference arises on the one hand because the neutron charge form factor is nonzero (leading to a small effect). On the other hand, the expressions (2.7a) contain terms which involve the proton and neutron magnetic form factors and these conspire to yield a much larger isovector than isoscalar contribution (see below). Since these G_M -effects are multiplied by $\eta_F^2 \approx 0.04$ – 0.08 , they tend to be suppressed; however, since they are multiplied by τ , they provide increasingly important contributions as the momentum transfer increases. The usual approach of including these (relativistic) corrections via the Darwin–Foldy term already breaks down for $q < 1$ GeV/c, as discussed in ref.¹⁵).

From (2.1) it clearly appears that the self-energy contribution actually results from the cancellation of two pieces associated, respectively, with the dressing of the hole and particle propagation: the larger term is the latter (at least for Λ_π not too small and q not too large, see sect. 4 and fig. 2). Therefore, $\Delta R_{(s.e.)}^{corr}$ depletes the nuclear response at low- ω , while enhancing it at high- ω , as it should in order to obey the sum rule requirement¹⁹). Worth noticing is that the contribution of the contact (δ) term of the OPEP cancels in (2.1), but for the momentum-dependence introduced by the vertex function Γ_π . When the latter is taken into account, as in our case, then an additional cancellation between the δ - and momentum-dependent contributions occurs. What remains is then a rather modest contribution (negative at small and positive at large ω , again for not too small Λ_π and not too large q), that rapidly decreases as q increases. This is illustrated in fig. 2 for $q = 300, 500$ and 1000 MeV/c in the longitudinal channel. For sake of illustration here and in the following we take $k_F = 225$ MeV/c (which roughly corresponds to the case of ^{12}C) and $\Lambda_\pi = 1300$ MeV, unless otherwise specified.

A similar situation also holds in the transverse channel, but for the well-known factor $Z\mu_p^2 + N\mu_n^2 = \frac{1}{2}A(\mu_p^2 + \mu_n^2) = \frac{1}{4}A(\mu_s^2 + \mu_v^2)$, which greatly suppresses the isoscalar contribution compared to the isovector contribution because of the fact that $(\mu_s/\mu_v)^2 \cong 0.035$.

An issue we wish to address in closing this section relates to the self-energy diagrams of higher-order in perturbation theory. Do they play an important role?

To answer this question at least partially, we may examine the longitudinal electromagnetic response in the Hartree–Fock approximation (actually here only the Fock approximation, since the Hartree contribution vanishes in nuclear matter, although we shall continue to use the label HF) and compare with the results displayed in fig. 2 to which we add the free response. The HF longitudinal response reads ¹⁵⁾

$$R_L^{HF}(q, \omega) = \frac{\xi_A}{2(2\pi)^3 \eta_F^3 m_N} f_L^2(Q^2) \int d\boldsymbol{\beta} \theta(\eta_F - \beta) \theta(|2\boldsymbol{\kappa} + \boldsymbol{\beta}| - \eta_F) \\ \times \delta \left\{ \lambda - |\boldsymbol{\kappa}^2 - \lambda^2| - \boldsymbol{\kappa} \cdot \boldsymbol{\beta} - [\tilde{\Sigma}_\pi^{(1)}(|\boldsymbol{\kappa} + \boldsymbol{\beta}|) - \tilde{\Sigma}_\pi^{(1)}(\beta)] \right\}, \quad (2.10)$$

where

$$\tilde{\Sigma}_\pi^{(1)}(\beta) = 3 \frac{f_\pi^2}{m_\pi^2} \frac{m_N^2}{\eta_F^2} \mathcal{G}(\beta) \quad (2.11)$$

is the pion self-energy.

In fig. 3 the longitudinal HF response is displayed for $q = 300, 500$ and 1000 MeV/c. Note that this response appears in a region of the (q, ω) –plane somewhat different from that where the RFG responses are nonzero, notwithstanding that each order of perturbation theory in the self-energy is actually nonzero precisely where the RFG is nonzero. The effect, however, is so small (at most a few MeV) that it can only be perceived at low- q , as seen in fig. 3. Remarkably, from this figure one realizes that the first-order response practically coincides with the HF result: the many-body framework apparently quells the strong interaction carried by the pion to the point of rendering quite accurate a first-order perturbative treatment of the OPEP, at least as far as the mean-field contributions are concerned. Accordingly, the impact of the pion self-energy contribution on the total charge response is rather mild and one observes some hardening for $q \approx 300$ MeV/c, which, however, disappears as q increases (see sect. 6).

3. Exchange contributions

In the 1p–1h sector of the nuclear response, in addition to the self–energy diagrams dealt with in the previous section, two further classes of Goldstone diagrams yield contributions in first–order perturbation theory: they are displayed in fig. 4 and are commonly referred to as *exchange diagrams*. In particular, in fig. 4a the 1p–1h state propagates forward in time (as in the so–called Tamm–Dancoff approximation (TDA)), whereas in fig. 4b a backward–going 1p–1h state occurs (as when one incorporates ground–state correlations in the RPA). The forward– and backward–going first–order perturbation theory contributions will be denoted F and B , respectively. The expressions for the associated contributions to the electromagnetic charge response read:

$$\begin{aligned}
\Delta R_{L(exch,F)}^{corr}(q, \omega) &= f_L^2(Q^2) \xi_A \frac{2}{k_F^3} \frac{f_\pi^2}{m_\pi^2} \\
&\times \int \frac{d\mathbf{k}_1}{(2\pi)^3} \frac{d\mathbf{k}_2}{(2\pi)^3} \theta(|\mathbf{k}_1 + \mathbf{q}| - k_F) \theta(k_F - k_1) \theta(|\mathbf{k}_2 + \mathbf{q}| - k_F) \theta(k_F - k_2) \\
&\times \Gamma_\pi(\mathbf{k}_1 + \mathbf{q}) \Gamma_\pi(\mathbf{k}_1) \frac{\delta(\omega - |Q^2|/2m_N - \mathbf{q} \cdot \mathbf{k}_1/m_N)}{\omega - |Q^2|/2m_N - \mathbf{q} \cdot \mathbf{k}_2/m_N} \frac{(\mathbf{k}_1 - \mathbf{k}_2)^2}{m_\pi^2 + (\mathbf{k}_1 - \mathbf{k}_2)^2} \quad (3.1a) \\
&= f_L^2(Q^2) \frac{1}{128\pi^4} \frac{\xi_A}{\eta_F \kappa^2} \frac{f_\pi^2}{m_N} \\
&\times \left\{ \theta(\eta_F - \kappa) \int_{k_1^<}^1 dk_1 \int_{|2\kappa/\eta_F - 1|}^1 dk_2 [\mathcal{N}_L(k_1, k_2; k_2, \psi_r) - \mathcal{N}_L(k_1, k_2; t, \psi_r)] \right. \\
&\quad \left. + \theta(2\kappa - \eta_F) \int_{k_1^<}^1 dk_1 \int_0^{k_2^>} dk_2 [\mathcal{N}_L(k_1, k_2; k_2, \psi_r) - \mathcal{N}_L(k_1, k_2; -k_2, \psi_r)] \right\} \quad (3.1b)
\end{aligned}$$

and

$$\begin{aligned}
\Delta R_{L(exch,B)}^{corr}(q, \omega) &= -f_L^2(Q^2) \xi_A \frac{2}{k_F^3} \frac{f_\pi^2}{m_\pi^2} \\
&\times \int \frac{d\mathbf{k}_1}{(2\pi)^3} \frac{d\mathbf{k}_2}{(2\pi)^3} \theta(|\mathbf{k}_1 + \mathbf{q}| - k_F) \theta(k_F - k_1) \theta(k_F - |\mathbf{k}_2 + \mathbf{q}|) \theta(k_2 - k_F) \\
&\times \Gamma_\pi(\mathbf{k}_1 + \mathbf{q}) \Gamma_\pi(\mathbf{k}_1) \frac{\delta(\omega - |Q^2|/2m_N - \mathbf{q} \cdot \mathbf{k}_1/m_N)}{\omega - |Q^2|/2m_N - \mathbf{q} \cdot \mathbf{k}_2/m_N} \frac{(\mathbf{k}_1 - \mathbf{k}_2)^2}{m_\pi^2 + (\mathbf{k}_1 - \mathbf{k}_2)^2} \quad (3.2a) \\
&= f_L^2(Q^2) \frac{1}{128\pi^4} \frac{\xi_A}{\eta_F \kappa^2} \frac{f_\pi^2}{m_N}
\end{aligned}$$

$$\begin{aligned} & \times \left\{ \theta(\eta_F - \kappa) \int_{\bar{k}_1^<}^{\bar{k}_1^>} dk_1 \int_{|2\kappa/\eta_F - 1|}^1 dk_2 [\mathcal{N}_L(k_1, k_2; k_2, -\bar{\psi}_r) - \mathcal{N}_L(k_1, k_2; t, -\bar{\psi}_r)] \right. \\ & \quad \left. + \theta(2\kappa - \eta_F) \int_{\bar{k}_1^<}^{\bar{k}_1^>} dk_1 \int_0^{\bar{k}_2^>} dk_2 [\mathcal{N}_L(k_1, k_2; k_2, -\bar{\psi}_r) - \mathcal{N}_L(k_1, k_2; -k_2, -\bar{\psi}_r)] \right\}, \end{aligned} \quad (3.2b)$$

respectively.

In the transverse channel we have instead

$$\begin{aligned} \Delta R_{T(exch,F)}^{corr}(q, \omega) &= f_T^2(Q^2) \xi_A \frac{2}{k_F^3} \frac{f_\pi^2}{m_\pi^2} \frac{1}{4m_N^2} (\mu_V^2 - 3\mu_S^2) \\ & \times \int \frac{d\mathbf{k}_1}{(2\pi)^3} \frac{d\mathbf{k}_2}{(2\pi)^3} \theta(|\mathbf{k}_1 + \mathbf{q}| - k_F) \theta(k_F - k_1) \theta(|\mathbf{k}_2 + \mathbf{q}| - k_F) \theta(k_F - k_2) \\ & \times \Gamma_\pi(\mathbf{k}_1 + \mathbf{q}) \Gamma_\pi(\mathbf{k}_1) \frac{\delta(\omega - |Q^2|/2m_N - \mathbf{q} \cdot \mathbf{k}_1/m_N)}{\omega - |Q^2|/2m_N - \mathbf{q} \cdot \mathbf{k}_2/m_N} \frac{[\mathbf{q} \cdot (\mathbf{k}_1 - \mathbf{k}_2)]^2}{m_\pi^2 + (\mathbf{k}_1 - \mathbf{k}_2)^2} \quad (3.3a) \\ & = f_T^2(Q^2) \frac{1}{128\pi^4} \frac{\xi_A}{\eta_F} \frac{f_\pi^2}{m_N} (\mu_V^2 - 3\mu_S^2) \\ & \times \left\{ \theta(\eta_F - \kappa) \int_{k_1^<}^1 dk_1 \int_{|2\kappa/\eta_F - 1|}^1 dk_2 [\mathcal{N}_T(k_1, k_2; k_2, \psi_r) - \mathcal{N}_T(k_1, k_2; t, \psi_r)] \right. \\ & \quad \left. + \theta(2\kappa - \eta_F) \psi_r [\mathcal{H}_T(1, q_<) - \mathcal{H}_T(k_1^<, q_<)] \right\} \quad (3.3b) \end{aligned}$$

and

$$\begin{aligned} \Delta R_{T(exch,B)}^{corr}(q, \omega) &= -f_T^2(Q^2) \xi_A \frac{2}{k_F^3} \frac{f_\pi^2}{m_\pi^2} \frac{1}{4m_N^2} (\mu_V^2 - 3\mu_S^2) \\ & \times \int \frac{d\mathbf{k}_1}{(2\pi)^3} \frac{d\mathbf{k}_2}{(2\pi)^3} \theta(|\mathbf{k}_1 + \mathbf{q}| - k_F) \theta(k_F - k_1) \theta(k_F - |\mathbf{k}_2 + \mathbf{q}|) \theta(k_2 - k_F) \\ & \times \Gamma_\pi(\mathbf{k}_1 + \mathbf{q}) \Gamma_\pi(\mathbf{k}_1) \frac{\delta(\omega - |Q^2|/2m_N - \mathbf{q} \cdot \mathbf{k}_1/m_N)}{\omega - |Q^2|/2m_N - \mathbf{q} \cdot \mathbf{k}_2/m_N} \frac{[\mathbf{q} \cdot (\mathbf{k}_1 - \mathbf{k}_2)]^2}{m_\pi^2 + (\mathbf{k}_1 - \mathbf{k}_2)^2} \quad (3.4a) \\ & = f_T^2(Q^2) \frac{1}{128\pi^4} \frac{\xi_A}{\eta_F} \frac{f_\pi^2}{m_N} (\mu_V^2 - 3\mu_S^2) \\ & \times \left\{ \theta(\eta_F - \kappa) \int_{\bar{k}_1^<}^{\bar{k}_1^>} dk_1 \int_{|2\kappa/\eta_F - 1|}^1 dk_2 [-\mathcal{N}_T(-k_1, k_2; k_2, -\bar{\psi}_r) + \mathcal{N}_T(-k_1, k_2; t, -\bar{\psi}_r)] \right. \\ & \quad \left. - \theta(2\kappa - \eta_F) \bar{\psi}_r [\mathcal{H}_T(\bar{k}_1^>, q_<) - \mathcal{H}_T(\bar{k}_1^<, q_<)] \right\}. \quad (3.4b) \end{aligned}$$

In (3.1-4) the following quantities have been introduced:

$$\begin{aligned}
t &= \frac{\eta_F^2 - 4\kappa^2 - k_2^2/m_N^2}{4\eta_F\kappa} \\
k_1^< &= \begin{cases} \sqrt{1 - (2\kappa/\eta_F)^2 - 4\kappa\psi_r/\eta_F}, & \psi_r < 1 - 2\kappa/\eta_F \\ |\psi_r|, & \psi_r > 1 - 2\kappa/\eta_F \end{cases} \\
k_2^> &= \begin{cases} 2\kappa/\eta_F - 1, & \kappa < \eta_F \\ 1, & \kappa > \eta_F \end{cases} \\
\bar{k}_1^< &= \begin{cases} 1, & \psi_r < 1 - 2\kappa/\eta_F \rightarrow \bar{\psi}_r < 1 \\ \bar{\psi}_r, & \psi_r > 1 - 2\kappa/\eta_F \rightarrow \bar{\psi}_r > 1 \end{cases} \\
\bar{k}_1^> &= \sqrt{1 + (2\kappa/\eta_F)^2 + 4\kappa\bar{\psi}_r/\eta_F} = \sqrt{1 - (2\kappa/\eta_F)^2 + 4\kappa\bar{\psi}_r/\eta_F} \\
q < &= \begin{cases} 2\kappa/\eta_F, & \kappa < \eta_F \\ 2, & \kappa > \eta_F \end{cases}
\end{aligned} \tag{3.5}$$

whereas the functions $\mathcal{N}_{L,T}$ and \mathcal{H}_T are defined in Appendix B. Note that while we have been able to express the exchange contribution to the transverse response analytically, at least when $\kappa > \eta_F$, this turned out not to be possible in the case of the longitudinal channel.

A few remarks should be made concerning the above expressions. First, notice that all of the exchange contributions are different from zero in the response region of the RFG, *vanishing*, however, on its boundaries (at variance with the self-energy case). Second, we note that the exchange contributions can be split, as in the self-energy case, into isoscalar and isovector components. By carrying out the pertinent traces over the Pauli matrices (see Appendix A), in this case it is found that in the $\tau = 0$ channel the pion exchange correlations are *three times stronger* than in $\tau = 1$ case, being moreover of opposite sign in the two isospin channels. Finally, performing a similar analysis in spin space (see Appendix A), one finds that the tensor interaction S_{12} *does not contribute* to the charge response, but only to the spin response.

The results of our calculation of the exchange contribution to the nuclear responses are displayed in fig. 5, where both the forward- and backward-going exchange contributions to the charge response are displayed for $q = 300, 500$ and 1000

MeV/c, and in fig. 6, where the same is done for the spin response. A feature common to all of the results we present here is immediately apparent: strength is removed by the pionic exchange correlations from the low-energy side of the nuclear responses and added to the high-energy side. This *hardening* effect ²⁰⁾ goes in parallel with the one induced by the self-energy correlations discussed in sect. 2, which however was somewhat milder. As a consequence both charge and spin responses turn out to be hardened with respect to the corresponding free RFG responses.

The hardening of the charge response arises from the strong *repulsive* character of the exchange isoscalar pionic correlations, which are overwhelmingly carried by the δ -component of the OPEP (the *attractive* isovector correlations for zero-range contributions are three times weaker, as mentioned above). The finite-range central interaction of the OPEP, while attractive is not very effective and only modestly reduces the impact of the δ -force on the charge response. Also worth mentioning is that the role of δ -contributions is clearly apparent in the F -term and becomes dominant in the B -term.

In the spin channel the isoscalar correlations are dramatically suppressed relative to the isovector correlations by the factor $(\mu_S/\mu_V)^2 \cong 0.035$. In contrast to the situation for the charge case, the isovector central correlations are now of a *repulsive* character (see Appendix A for the derivation of the related spinology), leading again to a hardening of the spin response due to the action of the δ -force. The latter, however, is counteracted here not only by the attractive momentum-dependent central interaction, but by the tensor force as well. Accordingly, the amount of hardening of the spin response is somewhat moderated with respect to the charge case.

Interestingly, in the spin channel the δ -dominance is restricted only to the F -diagram, since the largest contribution to the B -diagram is in fact provided by the tensor force — and the latter is known to be quite efficient in providing ground-state correlations. This explains why the contribution from the B -diagram survives in the spin channel up to larger momenta than in the charge channel. With regard to the momentum-dependence of the exchange contributions to the nuclear

responses, one should also notice that the B -term falls faster than the F -term as the momentum transfer increases because, as seen from (3.1b), (3.2b), (3.3b) and (3.4b), the integrals involved in the former span a higher momentum range than those in the latter. As a consequence, the backward-going responses are cut-off more rapidly by Γ_π than are the forward-going ones.

In summary, from the above analysis the following observations can be made:

- a) the exchange diagrams of fig. 4 are the ones that contribute most strongly to the nuclear responses as first-order perturbations in the 1p-1h sector of the RFG;
- b) their magnitude and sign is to a large extent set by the δ -component of the OPEP.

This last finding will be addressed further in sect. 5 in connection with the discussion of the global gauge invariance of our theory.

A further aspect of our results worth noting is that the sum-rule requirement is indeed fulfilled by the F -contribution, whereas it is violated by the B -term.²¹⁾ Indeed, the former is characterized by a node in the response function (when expressed as a function of ω) located at the peak position of the RFG response. Here the very tiny discrepancies seen can be ascribed to the present approximate, although very good, treatment of the relativistic kinematics. In contrast, the latter never changes sign, always remaining negative. In this connection one sees that the hardening of the nuclear responses previously discussed clearly follows from the way the sum rule is fulfilled by the F -diagram. In addition a significant contribution to the hardening also comes from the B -diagram. In fact, because the (negative) B -term obeys the energy-weighted sum rule, it then reaches its minimum somewhat before the peak of the free response.

To conclude this section we now explore the role played by exchange diagrams of order higher than one in the nuclear response. We do so for the charge longitudinal channel in the TDA, which amounts to accounting for all terms in the series shown in fig. 7. To achieve this, we employ the method of continued fractions²²⁾: in this

framework the polarization propagator becomes

$$\Pi_{L,\tau}^{\text{CF}}(\kappa, \lambda) = \frac{\Pi^0(\kappa, \lambda)}{1 + \Pi_{L,\tau}^{(1)}(\kappa, \lambda)/\Pi^0(\kappa, \lambda) + \dots}, \quad (3.6)$$

where $\Pi_{L,\tau}^{(1)}$ is the longitudinal first-order exchange contribution in the isospin channel τ and Π^0 the RFG propagator. The imaginary part of $\Pi_{L,\tau}^{(1)}$ is proportional to (3.1), whereas the real part is given in Appendix B.

We limit our attention to the first one of the infinite set of continued fractions embodied in (3.6), recalling that in the limit of a zero-range force the first iteration of (3.6) actually corresponds to the exact TDA solution. As previously discussed, since the δ -piece of the interaction plays the largest role in the charge response, we believe that the results displayed in fig. 8 are quite representative of the true TDA solution (of course, one should also take into account that in coordinate space Γ_π actually smears out the δ -function).

In order to characterize the results obtained here, in fig. 8 two cases are shown: i) the longitudinal TDA exchange response with a pure δ -interaction together with Γ_π and ii) adding to this the momentum-dependent forces as well. The latter, which corresponds to a weaker interaction, shows that the first-order result seems to be quite accurate even at $q = 500$ MeV/c. However, the degree of accuracy appears to be lower than in the corresponding self-energy case, in accord with the finding that the action of the pion is stronger in the exchange channel than in the self-energy channel.

4. Electromagnetic and weak neutral current responses

In the previous two sections we have set up all the ingredients needed to calculate not only the electromagnetic responses, but also the weak neutral current ones ^{23–27}) in first– (and as well in infinite–) order perturbation theory, within a framework where only the pion–induced correlations are taken into account. Indeed, by splitting the electromagnetic responses into their isospace components according to

$$R_L^{\text{em}} = R_L^{\text{em}}(\tau = 0) + R_L^{\text{em}}(\tau = 1) \quad (4.1a)$$

$$R_T^{\text{em}} = R_T^{\text{em}}(\tau = 0) + R_T^{\text{em}}(\tau = 1) \quad (4.1b)$$

and exploiting the conservation of the vector current (CVC) ²⁶), one easily obtains the weak neutral current longitudinal and transverse responses (as mentioned earlier, the axial–vector one is discussed in a companion paper ¹⁸)). They read

$$R_{AV}^L = a_A \left[\beta_V^{(0)} R_L^{\text{em}}(\tau = 0) + \beta_V^{(1)} R_L^{\text{em}}(\tau = 1) \right] \quad (4.2a)$$

$$R_{AV}^T = a_A \left[\beta_V^{(0)} R_T^{\text{em}}(\tau = 0) + \beta_V^{(1)} R_T^{\text{em}}(\tau = 1) \right], \quad (4.2b)$$

where $a_A = -1$ and the isoscalar and isovector weak neutral current couplings, $\beta_V^{(0)}$ and $\beta_V^{(1)}$, respectively, are fixed at tree–level in the standard model to the values

$$\beta_V^{(0)} = -2 \sin^2 \theta_W \approx -0.454 \quad (4.3a)$$

and

$$\beta_V^{(1)} = 1 - 2 \sin^2 \theta_W \approx 0.546, \quad (4.3b)$$

using $\sin^2 \theta_W \approx 0.227$ for the Weinberg angle.

To the contributions stemming from the correlations, those arising from the MEC should then be added. These have been thoroughly discussed within the context of 1p–1h excitations in ref. ¹⁵), where their contribution to the electromagnetic responses has been calculated including terms of order up to κ^2 in a non–relativistic reduction, where, in addition to spatial components, the time component of the MEC has also been kept in the evaluation. The ensuing violation of the continuity equation thus introduced has been checked in ref. ¹⁵) and found to be quite small.

The associated expressions in the weak neutral current sector are most easily obtained by recalling that the MEC for leading-order π -exchange are purely isovector in nature. Accordingly one simply obtains

$$R_{AV}^L(\text{MEC}) = a_A \beta_V^{(1)} R_L^{\text{em}}(\text{MEC}) \quad (4.4a)$$

$$R_{AV}^T(\text{MEC}) = a_A \beta_V^{(1)} R_T^{\text{em}}(\text{MEC}) \quad (4.4b).$$

In figs. 9 and 10 we now display the electromagnetic responses (longitudinal and transverse, respectively) at $q = 300, 500$ and 1000 MeV/c. We observe that in both channels a hardening of the responses occurs, particularly at low- q , essentially induced by the δ -force via the exchange diagram, as discussed at length in the previous section. Also in accord with the considerations developed there the shift in the maximum of the response is seen to be larger in the charge channel than in the spin one. As the momentum transfer increases, the amount of hardening decreases, since the pion-induced correlations weaken for large q . This behaviour partly reflects the role of the hadronic vertex function Γ_π in cutting out the high-momentum contributions to the response functions and partly reflects the constraints arising from the many-body system itself, which appear through the mismatch between the single-particle wave functions of the particle-hole pairs involved in the intermediate states of the polarization propagators considered here.

Clearly, a unique way to assess the impact of correlations on the response functions does not exist. Among several different possibilities, we have chosen to focus on the following two quantities: i) the shift $\Delta\omega_{\text{QEP}}$ of the peak position of the response with respect to the corresponding quantity in the RFG, which is displayed versus q in fig. 11 for both the longitudinal and transverse channels; ii) the correlated sum rule, *i.e.* the integral of the nuclear responses over ω , divided, in the longitudinal and transverse channels, by the functions

$$C_{L,T}(\tau, \psi_r; \eta_F) = \frac{\mathcal{N}}{4m_N \kappa} \times \begin{cases} \frac{\kappa^2}{\tau} [(1 + \tau)W_2(\tau) - W_1(\tau) + W_2(\tau)\Delta] & \text{for } L \\ 2W_1(\tau) + W_2(\tau)\Delta & \text{for } T \end{cases} \quad (4.5)$$

respectively. As shown in ref. ¹⁶⁾, this optimally separates the nucleonic physics from the many-body problem for the RFG. In (4.5)

$$W_1(\tau) = \tau G_M^2(\tau) \quad (4.6a)$$

$$W_2(\tau) = \frac{1}{1+\tau} (G_E^2(\tau) + \tau G_M^2(\tau)). \quad (4.6b)$$

and

$$\Delta = \frac{\tau}{\kappa^2} (1 - \psi_r^2) \xi_F \left\{ \kappa \sqrt{1 + \frac{1}{\tau}} + \frac{1}{3} (1 - \psi_r^2) \xi_F \right\}, \quad (4.7)$$

ψ_r being the scaling variable (2.6a) and $\xi_F = \sqrt{1 + \eta_F^2} - 1$. We recall that with these definitions we actually integrate the part of the responses which scales. So it should not be surprising that the asymptotic value, namely one, is reached for momenta much larger than $2k_F$. The normalized sum rules are reported versus q in figs. 12a and 12b, again for both channels. We recall that the sum rules reflect only the roles of the backward-going correlations and of the MEC.

In accord with the discussion of sect. 3, from these figures one first infers that the correlations persist up to larger momenta in the longitudinal channel than in the transverse one; secondly, one sees that the MEC contribution, although not large, is significant, rather constant with q and acts in a way that depletes the sum rule. Finally inspection of the responses shows that in the longitudinal channel the B -terms decrease in magnitude with q more rapidly than the correlations stemming from the F -diagram, whereas in the transverse channel the opposite occurs — a clear signature of the action of the tensor force.

Turning now to a discussion of the weak neutral current responses, we first consider the charge channel. To understand the nature of the problem, we display in fig. 13 the longitudinal nuclear response at $q = 300$ MeV/c separated into $\tau = 0$ and $\tau = 1$ channels. There we see that the action of the pion is very important in hardening the isoscalar channel, while rather gentle in softening the isovector one. Note that the effect of the longitudinal electromagnetic form factors in contributing to this imbalance is negligible at this low momentum transfer. When we combine the two isospin contributions with weighting factors $\beta_V^{(0)}$ and $\beta_V^{(1)}$ according to (4.2a)

to get the longitudinal weak neutral current response we obtain the result shown in fig. 14. While the magnitude of free response is quite small, the pion-correlated one is dramatically modified. The origin of this effect was discussed previously in ref. ²⁶⁾ (see also refs. ^{27,24)}), where it was noted that the delicate cancellation that leads to the suppression of R_{AV}^L in the RFG model is broken when the isoscalar and isovector channels are correlated differently. This heightened sensitivity to isospin correlation effects opens a new window on nuclear physics in the quasielastic region, possibly offering the unique possibility of separating the two isospin channels, something that is impossible to achieve with parity-conserving electron scattering alone. It can, however, be brought to light using parity-violating electron scattering, as discussed in ref. ²⁶⁾, where it is shown that isospin correlations have dramatic consequences for the forward-angle parity-violating asymmetry over a range of momentum transfers persisting up to $q \approx 500$ MeV/c.

In concluding this section, we return to touch on the transverse weak neutral current response R_{AV}^T : here the factor μ_s^2 suppresses the isoscalar contributions and consequently no sensitive cancellation occurs in the free RFG model to be broken by isospin correlations, in contrast to the case of R_{AV}^L . Accordingly, the prediction for R_{AV}^T is very simple: it is approximately half ($\beta_V^{(1)} = 0.546$) of the corresponding electromagnetic response (of course in (4.2a,b) the electromagnetic and weak neutral current coupling constants are not included). These observations were critical in our earlier work ²⁶⁾ where the insensitivity of the transverse response functions to isospin correlation effects allowed us to suggest backward-angle parity-violating quasielastic electron scattering as a tool to probe the single-nucleon form factors themselves. We shall return to quantify the roles played by pionic effects in this situation in ref. ¹⁸⁾.

5. Λ_π and k_F dependence of the nuclear responses

In this section we explore the significance of the roles played by the two parameters on which our calculation of the nuclear responses depends, namely Λ_π and k_F . We start by considering Λ_π , which incorporates some aspects of the short-range physics affecting the pionic correlations. To get a feeling for the distances involved in our problem, we consider the Fourier transform of Γ_π (see (2.5)), namely

$$\Gamma_\pi(r) = (\Lambda_\pi^2 - m_\pi^2) \frac{e^{-\Lambda_\pi r}}{4\pi r}. \quad (5.1)$$

The corresponding rms radius of the interaction region

$$\sqrt{\langle r^2 \rangle} = \left[\int d\mathbf{r} r^2 \Gamma_\pi(r) \right]^{1/2} = \frac{\sqrt{6}}{\Lambda_\pi} \sqrt{1 - (m_\pi/\Lambda_\pi)^2}, \quad (5.2)$$

is then easily obtained. Typical values of Λ_π are of order 1–2 GeV and hence $(m_\pi/\Lambda_\pi)^2 \sim 0.005$ –0.02, namely, only a small correction in (5.2). Accordingly the rms radius above is given to a good approximation by $\sqrt{6}/\Lambda_\pi$: for the value $\Lambda_\pi = 1300$ MeV used in most of this work, this yields an rms radius of 0.37 fm.

On the one hand, as seen in fig. 15 the sensitivity of the exchange contribution to variations in Λ_π appears to be quite mild. For sake of illustration we limit ourselves here and in the following to consideration only of the $q = 500$ MeV/c case and only of the longitudinal channel. The self-energy contribution, on the other hand, is more sensitive to changes in Λ_π , even reversing its behaviour in the energy variable for $\Lambda_\pi \approx 800$ MeV, reflecting the weakening of the particle self-energy with respect to that for the hole (fig. 15).

Somewhat in between the two situations discussed above is the behaviour versus Λ_π of the MEC (fig. 16). We may conclude that, owing to the major contribution to the nuclear responses arising from the exchange term, our pionic model is indeed affected, but only mildly, by the short-range physics embodied in the vertex function Γ_π for $\Lambda_\pi \gtrsim 1.3$ GeV.

Clearly, all of the short-range physics cannot be represented solely by the monopole form factor in (5.1). An adequate treatment of the short-range physics (presuming we continue to adopt a strategy of sticking to a mesonic model of the

nucleus) would require on the one hand the introduction of heavier mesons, and on the other the calculation of additional contributions (*e.g.*, the ladder diagrams) beyond the self-energy and exchange terms considered in the present approach. An alternative option would of course be to abandon the hadronic description of the short-range physics in favour of QCD degrees of freedom, but this has not yet proven to be tractable in the strong-coupling regime.

Next let us comment on how the continuity equation is affected by the vertex function Γ_π . In this connection we recall that the continuity equation in momentum space reads

$$i(\mathbf{p}_1 + \mathbf{p}_2) \cdot \mathbf{J}_{\text{MEC}}(\mathbf{p}_1, \mathbf{p}_2) = (\boldsymbol{\tau}_1 \times \boldsymbol{\tau}_2)^3 [V(\mathbf{p}_2) - V(\mathbf{p}_1)], \quad (5.3)$$

where \mathbf{p}_1 and $\mathbf{p}_2 = \mathbf{q} - \mathbf{p}_1$ are the momenta carried by the meson. For pointlike nucleons (5.3) is fulfilled by the OPEP (2.9) and by the longitudinal components of the pion-in-flight and contact currents:

$$\begin{aligned} (\mathbf{p}_1 + \mathbf{p}_2) \cdot \mathbf{J}_\pi(\mathbf{p}_1, \mathbf{p}_2) &= -i \frac{f_\pi^2}{m_\pi^2} (\boldsymbol{\tau}_1 \times \boldsymbol{\tau}_2)^3 \\ &\times \boldsymbol{\sigma}_1 \cdot \mathbf{p}_1 \boldsymbol{\sigma}_2 \cdot \mathbf{p}_2 \left(\frac{1}{m_\pi^2 + \mathbf{p}_1^2} - \frac{1}{m_\pi^2 + \mathbf{p}_2^2} \right) \end{aligned} \quad (5.4a)$$

and

$$\begin{aligned} (\mathbf{p}_1 + \mathbf{p}_2) \cdot \mathbf{J}_{\text{contact}}(\mathbf{p}_1, \mathbf{p}_2) &= -i \frac{f_\pi^2}{m_\pi^2} (\boldsymbol{\tau}_1 \times \boldsymbol{\tau}_2)^3 \\ &\times \left[\frac{\boldsymbol{\sigma}_1 \cdot \mathbf{p}_1 \boldsymbol{\sigma}_2 \cdot \mathbf{p}_2}{m_\pi^2 + \mathbf{p}_2^2} + \frac{\boldsymbol{\sigma}_1 \cdot \mathbf{p}_2 \boldsymbol{\sigma}_2 \cdot \mathbf{p}_2}{m_\pi^2 + \mathbf{p}_2^2} - \frac{\boldsymbol{\sigma}_1 \cdot \mathbf{p}_1 \boldsymbol{\sigma}_2 \cdot \mathbf{p}_1}{m_\pi^2 + \mathbf{p}_1^2} - \frac{\boldsymbol{\sigma}_1 \cdot \mathbf{p}_1 \boldsymbol{\sigma}_2 \cdot \mathbf{p}_2}{m_\pi^2 + \mathbf{p}_1^2} \right]. \end{aligned} \quad (5.4b)$$

It is of significance that the δ -piece of the OPEP does not contribute to (5.3), since its commutator with the charge operator vanishes. Accordingly, to keep or to drop the δ -term in the potential is of considerable relevance for the nuclear responses, but is irrelevant as far as the global gauge invariance of the theory is concerned. It becomes of importance, however, when the vertex function Γ_π is included. In fact, in order to satisfy the continuity equation in such a case new terms should be added to the pion-in-flight current, for example, the following¹⁷⁾

$$\mathbf{J}_\pi(\mathbf{p}_1, \mathbf{p}_2) = i \frac{f_\pi^2}{m_\pi^2} (\boldsymbol{\tau}_1 \times \boldsymbol{\tau}_2)^3 (\mathbf{p}_1 - \mathbf{p}_2) (\Lambda_\pi^2 - m_\pi^2)^2 \boldsymbol{\sigma}_1 \cdot \mathbf{p}_1 \boldsymbol{\sigma}_2 \cdot \mathbf{p}_2$$

$$\times \left\{ \frac{1}{(\Lambda_\pi^2 + \mathbf{p}_1^2)} \left[\frac{1}{(\Lambda_\pi^2 + \mathbf{p}_1^2)(m_\pi^2 + \mathbf{p}_1^2)} + \frac{1}{(m_\pi^2 + \mathbf{p}_1^2)(m_\pi^2 + \mathbf{p}_2^2)} + \frac{1}{(\Lambda_\pi^2 + \mathbf{p}_2^2)(m_\pi^2 + \mathbf{p}_2^2)} \right] \frac{1}{(\Lambda_\pi^2 + \mathbf{p}_2^2)} \right\}. \quad (5.5)$$

In contrast, no modifications are necessary but for the multiplicative factor Γ_π^2 occurring in the contact (or pair) term, this being the one contributed to by the δ (smeared) piece of the OPEP. The first and the third terms on the right-hand side of (5.5) may be viewed as describing the coupling of the virtual photon to a fictitious particle carrying the same quantum numbers as the pion, which thus underlies the microscopic description of Γ_π . We have numerically checked the importance of these additional terms and found them quite small (about 10% of the pion-in-flight contribution at the peak of the latter in the charge channel and about 20% in the spin channel under the same conditions and so yielding less than 1% corrections to the total responses). This occurs unless Λ_π becomes unreasonably small — say a few hundred MeV, a range of values that not only can hardly be accepted, but is also such as to render the overall pionic corrections to the RFG responses quite negligible. It remains however to be verified whether the above findings are still valid in the 2p-2h sector or when mesons heavier than the pion are brought into play.

As a general remark we note that, even in the presence of the vertex function Γ_π , it is still possible to drop the δ (smeared) piece of the OPEP while preserving gauge invariance via a suitable modification of the contact MEC current. Whether this should be done or not, however, is a question (as is the question about which value to choose for Λ_π) which should ultimately be answered by obtaining the best possible agreement with experiment. One might argue, however, that other mesons beyond the pion should be included in the formalism before attempting such a task: we intend to carry out this project in future work.

In closing this section, we wish to address the question of the k_F -dependence of the various pionic contributions to the nuclear responses. In our view, k_F is another parameter whose value should be set by comparing the theoretical predictions with experimental data. In figs. 17–20 we display the k_F -dependence of the various

contributions to the responses at $q = 500$ MeV/c divided by the functions

$$K_L(\tau, \psi_r; \eta_F) = \frac{3\mathcal{N}\xi_F}{4m_N\kappa\eta_F^3} \frac{\kappa^2}{\tau} [(1 + \tau)W_2(\tau) - W_1(\tau) + W_2(\tau)\Delta] \quad (5.6a)$$

and

$$K_T(\tau, \psi_r; \eta_F) = \frac{3\mathcal{N}\xi_F}{4m_N\kappa\eta_F^3} [2W_2(\tau) + W_2(\tau)\Delta] \quad (5.6b)$$

for the longitudinal and transverse channels, respectively. This permits us to focus only on the many-body part of the responses, and these reduced quantities will eventually scale for sufficiently high values of q . From fig. 17, where we display the self-energy contribution in the longitudinal electromagnetic channel, it is apparent that as k_F increases this contribution has a different behaviour at high- ω than it does at low- ω (*i.e.*, for scaling variable $\psi_r > 0$ and < 0 , respectively — see ref. ¹⁶) for a discussion of scaling in the RFG model). The net contribution to the total response, however, remains quite small.

More pronounced is the sensitivity of the exchange terms to variations of k_F , as illustrated in figs. 18a and b where the F and B longitudinal contributions are displayed. Indeed one sees that the F -term grows with k_F and for $|\psi_r| \simeq 0.8$ yields, in the longitudinal channel, a contribution as large as about 25% of the free response when $k_F = 250$ MeV/c (corresponding nuclei around ⁴⁰Ca). The B -term also grows with k_F , although its impact on the nuclear response reaches only about 15% of the free response. As previously found, the transverse channel appears to be somewhat less affected by the F -correlations (fig. 19). Also the B -correlations are more significant in the longitudinal channel than in the transverse one and, overall, they are felt less than the F -correlations. The above results are clearly in line with the dominance of the (smeared) δ -term in shaping the nuclear response as previously discussed.

As far as the MEC contributions (fig. 20) are concerned, their importance increases with k_F more rapidly in the transverse channel than in the longitudinal one. For small k_F (*i.e.*, either light nuclei or regions of low density) they affect the nuclear responses only modestly; in particular, for $k_F = 225$ MeV/c they contribute

only about 8% and 3% in the transverse and longitudinal channels, respectively. At higher k_F (250 MeV/c) they may yield a contribution larger than 10%.

As a last point to be discussed in this section, in parallel to the issue related to the impact of Λ_π on the continuity equation discussed above, we briefly address the question of how global electromagnetic gauge invariance is affected by the size of k_F . In this connection we consider the longitudinal/charge* nuclear response, since it is gauge invariance (through the continuity equation) that allows the replacement of the longitudinal component of the spatial electromagnetic current matrix elements with the charge matrix elements, or *vice versa*. We are not in a position to address this question at a quantitative level, because in the present paper, for sake of simplicity, we have ignored the convective current of the nucleon (only the dominant transverse spin current has been kept). We can, however, offer a simple guiding principle to help in recognizing whether gauge invariance is respected or not at the level of the many-body longitudinal/charge response: gauge invariance is fulfilled by a given Goldstone diagram contributing to the polarization propagator if its two electromagnetic vertices entail the *same* momentum flow. To understand what this means it helps to look at fig. 21. From the examples displayed there it is in fact apparent that, of the contributions calculated in the present paper, the self-energy diagrams respect gauge invariance, whereas the exchange ones do not. Analogously the antisymmetrized RPA (and even the ring diagrams alone, Fig. 21b) and ladder diagrams do not respect gauge invariance.

These differences may be understood by considering the general expression for

* The nomenclature used in studies of electroweak interactions with nuclei is not very good in this regard: what is referred to as the charge response in some places and the longitudinal response in others in fact usually involves both the $\mu = 0$ (charge) and $\mu = 3$ or z (longitudinal) projections of the 4-current. As we discuss in this section, the two aspects are related by the continuity equation when the current is conserved and thus it has become common practice to use the terms interchangeably. Throughout this article we have used the word “longitudinal” to refer to all $\mu = 0$ and 3 aspects of electroweak current matrix elements.

the response written as the sum of three terms, namely

$$\left(\frac{Q^2}{q^2}\right)^2 R_L(q, \omega) = R(QQ; q, \omega) - \frac{2\omega}{q} R_z(QJ; q, \omega) + \frac{\omega^2}{q^2} R_{zz}(JJ; q, \omega) . \quad (5.7)$$

In the above $R(QQ; q, \omega)$, $R_z(QJ; q, \omega)$ and $R_{zz}(JJ; q, \omega)$ refer to responses with two charge, one charge and one longitudinal current and two longitudinal current electromagnetic vertices, respectively. The z -component of the nucleonic current, in the leading order of the non-relativistic reduction, involves the convection current

$$J_z \propto (2\mathbf{k} + \mathbf{q}) \cdot \hat{\mathbf{q}} \quad (5.8)$$

and (2.1a), (3.1a) and (3.1b) contain integrals over \mathbf{k} (actually, in the last two cases the integration is over a vector called \mathbf{k}_2). Therefore, by exploiting the energy-conserving δ -function in the self-energy contribution (2.1a), it turns out that the current (5.8) can be expressed in this instance solely in terms of the external variables q and ω . Clearly, the same does not occur in the exchange contributions (3.1a) and (3.1b). Thus, via the continuity equation, for a translationally-invariant system treated at the level of the Hartree-Fock approximation, one has

$$R_z(QJ; q, \omega) = \frac{\omega}{q} R(QQ; q, \omega) \quad (5.9a)$$

and

$$R_{zz}(JJ; q, \omega) = \frac{\omega^2}{q^2} R(QQ; q, \omega) . \quad (5.9b)$$

Hence

$$R_L(q, \omega) = R(QQ; q, \omega) , \quad (5.10)$$

that is, $R_L(q, \omega)$ may be expressed entirely in terms of a single response function. This is not the case, for example, in the case of the antisymmetrized RPA. It follows that for large k_F , where the exchange contribution becomes relatively more important, it might even be necessary to account for the whole antisymmetrized RPA series, not to mention the ladder diagrams; in this instance serious consideration should be given to the gauge invariance of the theory.

6. Conclusions

Several motivations prompted the present study: first, we wished to investigate the inclusive charge and spin electromagnetic nuclear responses within a model that as much as possible respects Lorentz and gauge invariance building on the work in ref. ¹⁵). Secondly, our intention has been to explore, in parallel with the above, the weak neutral current responses (except for the axial–vector one whose analysis has been deferred to ref. ¹⁸). We have thus paved the way to the calculation of the asymmetry measured in parity–violating polarized electron scattering, which shall also be dealt with in ref. ¹⁸). Finally our purpose has been to investigate the degree to which a specific nuclear model based on nucleonic and mesonic degrees of freedom, when treated consistently at the level of the forces and currents, yields meaningful physical results over a relatively large range of momentum transfers even when the framework assumed is limited to first–order perturbation theory.

In this paper we have confined our attention to pionic effects, since the scale governing the range over which the currents and interactions are effective is especially large for the pion, being the lightest meson. Thus we have started with pionic effects as our initial focus; this approach serves as a paradigm for the treatment of heavier mesons as well. Indeed, our basic approach in which it has been possible to explore (albeit, within a specific model of the nucleus) the interplay of forces and currents has natural extensions where heavier mesons can be included as well.

Let us summarize our findings, beginning with the third of the motivations mentioned above. We have strived for consistency at two levels. The first one relates to the Feynman diagrams: of these we have retained all cases having one pionic line. This might appear insufficient for an adequate treatment of the forces and yet we have been able to show that results obtained in first–order perturbation theory are practically equivalent to those obtained in HF and RPA (especially to the first) when the force is carried by the pion. Accordingly, we are led to conjecture that even more sophisticated approaches, *e.g.*, RPA with HF–dressed fermionic propagators, would not be substantially different from those using first–order perturbation theory (at least in the 1p–1h sector). This outcome is of importance because it would clearly

be more difficult to account for pionic MEC diagrams with two or more pionic lines.

The second level of consistency relates to the continuity equation. Operationally this involves asking whether the δ -function part of the OPEP, given its crucial impact on the nuclear responses, affects the continuity equation or not. The answer is no for pointlike nucleons and pions, and yes for extended ones. Accordingly in the pointlike case one can, without affecting gauge invariance, drop the δ -function from the theory, arguing that short-range correlations will strongly quench its impact on the responses anyway.

On the other hand, some aspects of this quenching might be phenomenologically embedded in the factors Γ_π . But then the resulting *smeared* δ of the OPEP does contribute to the continuity equation. In such a case, to fulfill the latter one needs to modify the currents beyond the standard multiplicative factor Γ_π^2 , indeed by adding to the pion-in-flight current two extra pieces (that vanish when the mass term $\Lambda_\pi \rightarrow \infty$). Their contribution to the nuclear responses in the 1p-1h sector is, however, quite small, although it should be checked whether this remains true in the 2p-2h sector and for heavier mesons. A final option, not explored in the present paper, namely to drop the smeared δ of the OPEP, might deserve further study. In this case the restoration of gauge invariance can still be achieved by suitably modifying the contact current, namely, the only one contributed to by the δ .

Turning to the second motivation, we have found a striking correlation effect in the longitudinal weak neutral current response R_{AV}^L arising partly because the isovector and isoscalar components enter in R_{AV}^L with a particular sign combination (dictated by the standard model), thus yielding a delicate cancellation in the case of the free RFG, and partly because the $\tau = 0$ pionic correlations are much stronger, and of opposite sign, than the $\tau = 1$ ones. This outcome has an important bearing on the parity-violating polarized electron scattering asymmetry as discussed in ref. ²⁶⁾ (see also ref. ¹⁸⁾).

Finally, coming to the first motivation above, as far as the forces are concerned we have found that:

- i) the charge and the spin responses are both hardened (shifted to higher ω) with

respect to the RFG;

- ii) the hardening is greater in the longitudinal/charge channel (L) than in the spin channel (T);
- iii) the hardening fades away with increasing momentum transfer q (at ~ 1 GeV/c very little is left of it);
- iiii) the hardening is due to the Γ_π -modified δ -piece of the OPEP.

Regarding the currents (MEC) we have found that:

- i) for the nuclear responses in the 1p–1h sector the MEC contributions are small, but not negligible (about 10% of the nuclear response in the transverse channel and at most 5% in the longitudinal one);
- ii) their contribution to the sum rules, relative to the RFG contribution, is almost constant as a function of q ;
- iii) they always enter in such a way as to *decrease* the nuclear responses.

In conclusion, our investigation of pionic effects started in ref. ¹⁵⁾ and pursued in the present paper shows the importance of the role they play in determining the electromagnetic and weak neutral current quasielastic nuclear responses. Predictions regarding the q -dependent hardening of the longitudinal and transverse responses, R_L and R_T , as well as the dramatic modification of the longitudinal parity-violating response R_{AV}^L emerge from our results. The approach taken, namely through extensions to the relativistic Fermi gas model where Lorentz covariance and consistency in treating forces and currents can be maintained, allows us to explore the origin of these predictions in a relatively direct way. In particular, it has proven to be instructive to see how they arise from the interplay of exchange and self-energy contributions, of central and tensor pieces of the force and of isoscalar/isovector correlation effects. Naturally, more remains to be done before a complete (hadronic) picture will be attained — in future work we intend to explore the nature of the quasielastic responses when heavier mesons and summation schemes other than HF and RPA are incorporated using the same basic RFG-motivated framework.

Appendix A

In this Appendix we perform the spin and isospin traces entering into the self-energy and exchange diagrams of fig. 4.

a) Spin

The spin matrix element to be calculated in the first-order self-energy term is

$$S_{L,T}^{s.e.} = \sum_{s_a s_b s_c s_d} \langle s_a | \hat{O}_{L,T}^\sigma | s_b \rangle \langle s_b s_c | \hat{V}_\sigma | s_c s_d \rangle \langle s_d | \hat{O}_{L,T}^\sigma | s_a \rangle , \quad (A.1)$$

where $\hat{O}_{L,T}^\sigma$ is the spin operator associated to the γ NN vertex in the longitudinal or transverse channel, namely

$$\hat{O}_L^\sigma = \mathbb{1} \quad (A.2)$$

and

$$\hat{O}_T^\sigma = (\boldsymbol{\sigma} \times \hat{\mathbf{q}})_i , \quad (A.3)$$

where $i = 1, 2$ and $\mathbb{1}$ is the 2×2 unit matrix. In (A.1) \hat{V}_σ can be either the central $(\boldsymbol{\sigma}_1 \cdot \boldsymbol{\sigma}_2)$ or the tensor operator

$$S_{12}(\hat{\mathbf{k}}) = (3\hat{k}_i \hat{k}_j - \delta_{ij}) \sigma_{1i} \sigma_{2j} , \quad (A.4)$$

\mathbf{k} being the momentum carried by the pion. In the longitudinal channel one therefore gets

$$\begin{aligned} S_{L,central}^{s.e.} &= \sum_{s_a s_b s_c s_d} \langle s_a | \mathbb{1} | s_b \rangle \langle s_b | \sigma_i | s_c \rangle \langle s_c | \sigma_i | s_d \rangle \langle s_d | \mathbb{1} | s_a \rangle \\ &= Tr \{ \sigma_i \sigma_i \} = 6, \end{aligned} \quad (A.5)$$

(repeated indices are meant to be summed), whereas the tensor force does not contribute to the longitudinal self-energy:

$$\begin{aligned} S_{L,tensor}^{s.e.} &= \sum_{s_a s_b s_c s_d} \langle s_a | \mathbb{1} | s_b \rangle \langle s_b | \sigma_i | s_c \rangle \langle s_c | \sigma_j | s_d \rangle \langle s_d | \mathbb{1} | s_a \rangle (3\hat{k}_i \hat{k}_j - \delta_{ij}) \\ &= Tr \{ \sigma_i \sigma_j \} (3\hat{k}_i \hat{k}_j - \delta_{ij}) = 0. \end{aligned} \quad (A.6)$$

In the transverse channel the central part of the pion–exchange potential yields

$$\begin{aligned}
 S_{T,central}^{s.e.} &= \sum_{s_a s_b s_c s_d} \langle s_a | (\boldsymbol{\sigma} \times \hat{\mathbf{q}})_k | s_b \rangle \langle s_b | \sigma_i | s_c \rangle \langle s_c | \sigma_i | s_d \rangle \langle s_d | (\boldsymbol{\sigma} \times \hat{\mathbf{q}})_k | s_a \rangle \\
 &= \varepsilon_{klm} \varepsilon_{krs} \hat{q}_m \hat{q}_s \text{Tr} \{ \sigma_l \sigma_i \sigma_i \sigma_r \} = 12.
 \end{aligned} \tag{A.7}$$

Note that, in accord with the (A.3) for \hat{O}_T^σ , although the sum over k in (A.7) should be restricted to $k = 1, 2$, it is unnecessary to take all of the components, since

$$(\boldsymbol{\sigma} \times \hat{\mathbf{q}}) \cdot (\boldsymbol{\sigma} \times \hat{\mathbf{q}}) = (\boldsymbol{\sigma} \times \hat{\mathbf{q}})_1 (\boldsymbol{\sigma} \times \hat{\mathbf{q}})_1 + (\boldsymbol{\sigma} \times \hat{\mathbf{q}})_2 (\boldsymbol{\sigma} \times \hat{\mathbf{q}})_2 \tag{A.8}$$

if \mathbf{q} points along the z -axis. Using similar arguments one can show that the tensor interaction gives no contribution to the self–energy in the transverse channel:

$$\begin{aligned}
 S_{T,tensor}^{s.e.} &= \\
 &\sum_{s_a s_b s_c s_d} \langle s_a | (\boldsymbol{\sigma} \times \hat{\mathbf{q}})_k | s_b \rangle \langle s_b | \sigma_i | s_c \rangle \langle s_c | \sigma_j | s_d \rangle \langle s_d | (\boldsymbol{\sigma} \times \hat{\mathbf{q}})_k | s_a \rangle (3\hat{k}_i \hat{k}_j - \delta_{ij}) \\
 &= \varepsilon_{klm} \varepsilon_{krs} \hat{q}_m \hat{q}_s (3\hat{k}_i \hat{k}_j - \delta_{ij}) \text{Tr} \{ \sigma_l \sigma_i \sigma_j \sigma_r \} = 0,
 \end{aligned} \tag{A.9}$$

having used the property

$$\text{Tr} \{ \sigma_l \sigma_i \sigma_j \sigma_r \} = 2 (\delta_{li} \delta_{jr} + \delta_{lr} \delta_{ij} - \delta_{lj} \delta_{ir}). \tag{A.10}$$

Let us now consider the exchange term. The spin factor is then given by:

$$S_{L,T}^{exch} = \sum_{s_a s_b s_c s_d} \langle s_a | \hat{O}_{L,T}^\sigma | s_b \rangle \langle s_b s_d | \hat{V}_\sigma | s_c s_a \rangle \langle s_c | \hat{O}_{L,T}^\sigma | s_d \rangle. \tag{A.11}$$

In the longitudinal response this leads to

$$\begin{aligned}
 S_{L,central}^{exch} &= \sum_{s_a s_b s_c s_d} \langle s_a | \mathbb{1} | s_b \rangle \langle s_b | \sigma_i | s_c \rangle \langle s_d | \sigma_i | s_a \rangle \langle s_c | \mathbb{1} | s_d \rangle \\
 &= \text{Tr} \{ \sigma_i \sigma_i \} = 6
 \end{aligned} \tag{A.12}$$

and

$$\begin{aligned}
S_{L,tensor}^{exch} &= \sum_{s_a s_b s_c s_d} \langle s_a | \mathbb{1} | s_b \rangle \langle s_b | \sigma_i | s_c \rangle \langle s_d | \sigma_j | s_a \rangle \langle s_c | \mathbb{1} | s_d \rangle (3\hat{k}_i \hat{k}_j - \delta_{ij}) \\
&= Tr \{ \sigma_i \sigma_j \} (3\hat{k}_i \hat{k}_j - \delta_{ij}) = 0.
\end{aligned} \tag{A.13}$$

In the transverse response

$$\begin{aligned}
S_{T,central}^{exch} &= \sum_{s_a s_b s_c s_d} \langle s_a | (\boldsymbol{\sigma} \times \hat{\mathbf{q}})_k | s_b \rangle \langle s_b | \sigma_i | s_c \rangle \langle s_d | \sigma_i | s_a \rangle \langle s_c | (\boldsymbol{\sigma} \times \hat{\mathbf{q}})_k | s_d \rangle \\
&= \varepsilon_{klm} \varepsilon_{krs} \hat{q}_m \hat{q}_s Tr \{ \sigma_l \sigma_i \sigma_r \sigma_i \} = -4
\end{aligned} \tag{A.14}$$

and

$$\begin{aligned}
S_{T,tensor}^{exch} &= \\
&\sum_{s_a s_b s_c s_d} \langle s_a | (\boldsymbol{\sigma} \times \hat{\mathbf{q}})_k | s_b \rangle \langle s_b | \sigma_i | s_c \rangle \langle s_d | \sigma_j | s_a \rangle \langle s_c | (\boldsymbol{\sigma} \times \hat{\mathbf{q}})_k | s_d \rangle (3\hat{k}_i \hat{k}_j - \delta_{ij}) \\
&= \varepsilon_{klm} \varepsilon_{krs} \hat{q}_m \hat{q}_s (3\hat{k}_i \hat{k}_j - \delta_{ij}) Tr \{ \sigma_l \sigma_i \sigma_r \sigma_j \} = 4 \left[1 - 3 (\hat{\mathbf{k}} \cdot \hat{\mathbf{q}})^2 \right].
\end{aligned} \tag{A.15}$$

Note that the transverse channel is the only one where the tensor interaction gives a nonzero contribution, namely, via the exchange diagram.

b) Isospin

Let us now consider the isospin matrix elements involved in the self-energy and exchange terms. At the γ NN vertex we have both isoscalar and isovector (z -component) dependences: for example, the charge and convection current operators involve

$$\hat{O} = \frac{\mathbb{1} + \tau_z}{2} \tag{A.16a}$$

and the magnetization operator involves

$$\begin{aligned}
\hat{O} &= \left[\frac{\mathbb{1} + \tau_z}{2} \right] \mu_p + \left[\frac{\mathbb{1} - \tau_z}{2} \right] \mu_n \\
&= \mu_s \frac{\mathbb{1}}{2} + \mu_v \frac{\tau_z}{2}.
\end{aligned} \tag{A.16b}$$

Here the $\mathbb{1}$ -terms give rise to the isoscalar ($\tau = 0$) particle-hole responses, while the τ_z -terms correspond to the isovector ($\tau = 1$) ones.

The self-energy diagram takes equal contributions from the traces in the two isospin channels:

$$\begin{aligned} T^{s.e.}(\tau = 0) &= \frac{1}{4} \sum_{t_a t_b t_c t_d} \langle t_a | \mathbb{1} | t_b \rangle \langle t_b t_c | \tau_1 \cdot \tau_2 | t_c t_d \rangle \langle t_d | \mathbb{1} | t_a \rangle \\ &= \frac{1}{4} \text{Tr} \{ \tau_i \tau_i \} = \frac{3}{2} \end{aligned} \quad (\text{A.17})$$

and

$$\begin{aligned} T^{s.e.}(\tau = 1) &= \frac{1}{4} \sum_{t_a t_b t_c t_d} \langle t_a | \tau_z | t_b \rangle \langle t_b t_c | \tau_1 \cdot \tau_2 | t_c t_d \rangle \langle t_d | \tau_z | t_a \rangle \\ &= \frac{1}{4} \text{Tr} \{ \tau_z \tau_i \tau_i \tau_z \} = \frac{3}{2}. \end{aligned} \quad (\text{A.18})$$

In contrast, in the exchange term the isoscalar and the isovector traces are different in magnitude and sign. They read

$$\begin{aligned} T^{exch}(\tau = 0) &= \frac{1}{4} \sum_{t_a t_b t_c t_d} \langle t_a | \mathbb{1} | t_b \rangle \langle t_b t_d | \tau_1 \cdot \tau_2 | t_c t_a \rangle \langle t_c | \mathbb{1} | t_d \rangle \\ &= \frac{1}{4} \text{Tr} \{ \tau_i \tau_i \} = \frac{3}{2} \end{aligned} \quad (\text{A.19})$$

and

$$\begin{aligned} T^{exch}(\tau = 1) &= \frac{1}{4} \sum_{t_a t_b t_c t_d} \langle t_a | \tau_z | t_b \rangle \langle t_b t_d | \tau_1 \cdot \tau_2 | t_c t_a \rangle \langle t_c | \tau_z | t_d \rangle \\ &= \frac{1}{4} \text{Tr} \{ \tau_z \tau_i \tau_i \tau_z \} = -\frac{1}{2}. \end{aligned} \quad (\text{A.20})$$

Finally let us consider the n^{th} -order exchange diagram, which contains n pionic lines inside one bubble as in fig. 7. The corresponding isospin factor reads

$$T_n^{exch} = \text{Tr} \{ \hat{O}^\tau \tau_{i1} \tau_{i2} \dots \tau_{in-1} \tau_{in} \hat{O}^\tau \tau_{in} \tau_{in-1} \dots \tau_{i2} \tau_{i1} \} \quad (\text{A.21})$$

and can be obtained from the first-order ones by recursion relations:

$$T_n^{exch}(\tau = 0) = \frac{3^n}{2} \quad (\text{A.22})$$

and

$$T_n^{exch}(\tau = 1) = -T_{n-1}^{exch}(\tau = 1) = \frac{(-1)^n}{2}. \quad (\text{A.23})$$

Appendix B

In this Appendix we first give the functions $\mathcal{N}_{L,T}$ and \mathcal{H}_T entering in the formulae (3.1-4) for the exchange contributions to the response. The first two functions are the following:

$$\begin{aligned} \mathcal{N}_L(k_1, k_2; z, \psi) &= \frac{\tilde{\lambda}_\pi^2}{\tilde{m}_\pi^2} (\tilde{\lambda}_\pi^2 - \tilde{m}_\pi^2) \frac{4k_1 k_2}{\mathcal{F}_{\lambda_\pi}^2(k_1, k_2; \psi, \psi)} \\ &\times \left\{ \frac{\tilde{\lambda}_\pi^2 + k_1^2 + k_2^2 - 2\psi^2}{\mathcal{F}_{\lambda_\pi}^2(k_1, k_2; \psi, \psi)} \left[\ln |\mathcal{G}_{\lambda_\pi}(k_1, k_2; z, \psi)| - \ln |\psi - z| \right] + 1 \right. \\ &\left. - \frac{(\tilde{\lambda}_\pi^2 + k_1^2 + k_2^2)^3 - 4k_1^2 k_2^2 (\tilde{\lambda}_\pi^2 + k_1^2 + k_2^2) - 4\psi^2 k_2^2 (\tilde{\lambda}_\pi^2 - k_1^2 + k_2^2) - 4\psi z k_1^2 (\tilde{\lambda}_\pi^2 + k_1^2 - k_2^2)}{[(\tilde{\lambda}_\pi^2 + k_1^2 + k_2^2)^2 - 4k_1^2 k_2^2] \mathcal{F}_{\lambda_\pi}^2(k_1, k_2; z, \psi)} \right\} \\ &+ \mathcal{M}_L^{\tilde{m}_\pi}(k_1, k_2; z, \psi) - \mathcal{M}_L^{\tilde{\lambda}_\pi}(k_1, k_2; z, \psi) \end{aligned} \quad (B.1)$$

and

$$\begin{aligned} \mathcal{N}_T(k_1, k_2; z, \psi) &= -\frac{(\tilde{\lambda}_\pi^2 - \tilde{m}_\pi^2)}{\tilde{m}_\pi^2} \frac{k_2}{k_1} \left\{ \frac{\psi}{k_1} \ln |2k_1^2 z - \psi(\tilde{\lambda}_\pi^2 + k_1^2 + k_2^2) + k_1 \mathcal{F}_{\lambda_\pi}(k_1, k_2; z, \psi)| \right. \\ &\left. + \frac{(\tilde{\lambda}_\pi^2 + k_1^2 + k_2^2)^3 - 4(\tilde{\lambda}_\pi^2 + k_1^2 + k_2^2)(k_1^2 k_2^2 + \tilde{\lambda}_\pi^2 \psi z) + 4\psi k_2^2(\psi - z)(\tilde{\lambda}_\pi^2 - k_1^2 + k_2^2)}{[(\tilde{\lambda}_\pi^2 + k_1^2 + k_2^2)^2 - 4k_1^2 k_2^2] \mathcal{F}_{\lambda_\pi}^2(k_1, k_2; z, \psi)} \right\} \\ &+ \mathcal{M}_T^{\tilde{m}_\pi}(k_1, k_2; z, \psi) - \mathcal{M}_T^{\tilde{\lambda}_\pi}(k_1, k_2; z, \psi), \end{aligned} \quad (B.2)$$

where

$$\begin{aligned} \mathcal{M}_L^\lambda(k_1, k_2; z, \psi) &= -\frac{4k_1 k_2}{\mathcal{F}_\lambda(k_1, k_2; \psi, \psi)} \left[\ln |\mathcal{G}_\lambda(k_1, k_2; z, \psi)| - \ln |\psi - z| \right] \\ \mathcal{M}_T^\lambda(k_1, k_2; z, \psi) &= -\frac{1}{\tilde{m}_\pi^2} \frac{k_2}{k_1} \left\{ \mathcal{F}_\lambda(k_1, k_2; z, \psi) \right. \\ &\quad \left. + \frac{\psi}{k_1} (\lambda^2 - k_1^2 + k_2^2) \ln |2k_1^2 z - \psi(\lambda^2 + k_1^2 + k_2^2) + k_1 \mathcal{F}_\lambda(k_1, k_2; z, \psi)| \right\} \\ \mathcal{G}_\lambda(k_1, k_2; z, \psi) &= 2\psi(\psi - z)(\lambda^2 - k_1^2 + k_2^2) + \mathcal{F}_\lambda(k_1, k_2; z, \psi) \mathcal{F}_\lambda(k_1, k_2; \psi, \psi) \\ &\quad + \mathcal{F}_\lambda^2(k_1, k_2; \psi, \psi) \\ \mathcal{F}_\lambda(k_1, k_2; z, \psi) &= \sqrt{(\lambda^2 + k_1^2 + k_2^2)^2 - 4\psi z(\lambda^2 + k_1^2 + k_2^2) + 4k_1^2 z^2 + 4k_2^2 \psi^2 - 4k_1^2 k_2^2}. \end{aligned} \quad (B.3)$$

We have also set $\tilde{\lambda}_\pi = \Lambda_\pi/k_F$ and $\tilde{m}_\pi = m_\pi/k_F$. The function \mathcal{H}_T , which gives in a fully analytic form the transverse exchange response at $q > 2k_F$, reads

$$\mathcal{H}_T(k, p) = -\frac{(\tilde{\lambda}_\pi^2 - \tilde{m}_\pi^2)}{\tilde{m}_\pi^2} \left\{ 2\tilde{\lambda}_\pi \left[\tan^{-1} \left(\frac{k+p-1}{\tilde{\lambda}_\pi} \right) - \tan^{-1} \left(\frac{k-p+1}{\tilde{\lambda}_\pi} \right) \right] \right\}$$

$$\begin{aligned}
& + \frac{1}{2k} [\tilde{\lambda}_\pi^2 - k^2 + (p-1)^2] \ln \left| \frac{\tilde{\lambda}_\pi^2 + (k-p+1)^2}{\tilde{\lambda}_\pi^2 + (k+p-1)^2} \right| \Bigg\} \\
& + \mathcal{T}^{\tilde{m}_\pi}(k, p) - \mathcal{T}^{\tilde{\lambda}_\pi}(k, p), \tag{B.4}
\end{aligned}$$

where

$$\begin{aligned}
\mathcal{T}^\lambda(k, p) = & \frac{1}{\tilde{m}_\pi^2} \left\{ \frac{k^2}{3}(p-1) + \frac{4}{3}\lambda^3 \left[\tan^{-1} \left(\frac{k-p+1}{\lambda} \right) - \tan^{-1} \left(\frac{k+p-1}{\lambda} \right) \right] \right. \\
& + \frac{1}{12k} [k^4 - 6k^2(1-\lambda^2) - 3(1+\lambda^2)^2 + 12p(\lambda^2 + k^2 + 1) \\
& \quad \left. - 6p^2(\lambda^2 + k^2 + 3) - 3p^3(p-4)] \ln \left| \frac{\tilde{\lambda}_\pi^2 + (k-p+1)^2}{\tilde{\lambda}_\pi^2 + (k+p-1)^2} \right| \right. \\
& \left. + \frac{2}{3}(p-1)^3 \ln \left| [\tilde{\lambda}_\pi^2 + (k-p+1)^2][\tilde{\lambda}_\pi^2 + (k+p-1)^2] \right| \right\}. \tag{B.5}
\end{aligned}$$

Finally, we give the expression for the real part of the first-order forward-going (F) longitudinal polarization propagator (see eq. (3.6)), limited for simplicity to the case $q > 2k_F$:

$$\begin{aligned}
\text{Re}\Pi_{L,\tau}^{(1)F}(q, \omega) = & \xi_A \frac{1}{k_F^3} \frac{f_\pi^2}{m_\pi^2} \mathcal{I}_\tau \int \frac{d\mathbf{k}_1}{(2\pi)^3} \frac{d\mathbf{k}_2}{(2\pi)^3} \theta(k_F - k_1) \theta(k_F - k_2) \frac{(\mathbf{k}_1 - \mathbf{k}_2)^2}{m_\pi^2 + (\mathbf{k}_1 - \mathbf{k}_2)^2} \\
& \times \left[\frac{1}{\omega - |Q^2|/2m_N - \mathbf{q} \cdot \mathbf{k}_1/m_N} \frac{1}{\omega - |Q^2|/2m_N - \mathbf{q} \cdot \mathbf{k}_2/m_N} \right. \\
& \left. - \delta(\omega - |Q^2|/2m_N - \mathbf{q} \cdot \mathbf{k}_1/m_N) \delta(\omega - |Q^2|/2m_N - \mathbf{q} \cdot \mathbf{k}_2/m_N) \right] \tag{B.6a}
\end{aligned}$$

$$\begin{aligned}
& = \frac{1}{128\pi^4} \frac{\xi_A}{\eta_F \kappa^2} \frac{f_\pi^2}{m_N} \mathcal{I}_\tau \\
& \times \left\{ \int_0^1 dk_1 \int_{-1}^1 dx_1 \int_0^1 dk_2 [\mathcal{P}(k_1, k_2; k_1 x_1, \psi_r) - \mathcal{P}(k_1, -k_2; k_1 x_1, \psi_r)] \right. \\
& \quad + \pi^2 \int_{|\psi_r|}^1 dk k \ln \left| \frac{1 + \tilde{m}_\pi^2 - k^2 + \sqrt{(1 - \tilde{m}_\pi^2 - k^2)^2 + 4\tilde{m}_\pi^2(1 - \psi_r^2)}}{1 + \tilde{\lambda}_\pi^2 - k^2 + \sqrt{(1 - \tilde{\lambda}_\pi^2 - k^2)^2 + 4\tilde{\lambda}_\pi^2(1 - \psi_r^2)}} \right| \\
& \quad - \frac{\tilde{\lambda}_\pi^2 - \tilde{m}_\pi^2}{\tilde{m}_\pi^2} \frac{\pi^2}{4} (\tilde{\lambda}_\pi^2 + 2 - 2\psi_r^2 - \tilde{\lambda}_\pi \sqrt{\tilde{\lambda}_\pi^2 + 4 - 4\psi_r^2}) \\
& \quad \left. - \frac{\pi^2}{2} (1 - \psi_r^2) \ln \left(\frac{\tilde{m}_\pi^2}{\tilde{\lambda}_\pi^2} \right) \right\}, \tag{B.6b}
\end{aligned}$$

where $\mathcal{I}_{\tau=0} = 3/2$, $\mathcal{I}_{\tau=1} = -1/2$,

$$\begin{aligned} \mathcal{P}(k_1, k_2; y_1, \psi) = & \frac{\tilde{\lambda}_\pi^2}{\tilde{m}_\pi^2} (\tilde{\lambda}_\pi^2 - \tilde{m}_\pi^2) \left\{ \frac{-1}{\psi - y_1} \frac{4k_1^2 k_2^2 [y_1(\tilde{\lambda}_\pi^2 + k_1^2 + k_2^2) - 2k_1^2 \psi]}{[(\tilde{\lambda}_\pi^2 + k_1^2 + k_2^2)^2 - 4k_1^2 k_2^2] \mathcal{F}_{\lambda_\pi}^2(k_1, k_2; y_1, \psi)} \right. \\ & \left. - \frac{\tilde{\lambda}_\pi^2 + k_1^2 + k_2^2 - 2y_1 \psi}{\mathcal{F}_{\lambda_\pi}^2(k_1, k_2; y_1, \psi)} \mathcal{Q}^{\tilde{\lambda}_\pi}(k_1, k_2; y_1, \psi) \right\} \\ & + \mathcal{Q}^{\tilde{m}_\pi}(k_1, k_2; y_1, \psi) - \mathcal{Q}^{\tilde{\lambda}_\pi}(k_1, k_2; y_1, \psi) \end{aligned} \quad (B.7a)$$

and

$$\begin{aligned} \mathcal{Q}^\lambda(k_1, k_2; y_1, \psi) = & -\frac{k_1^2 k_2}{\psi - y_1} \frac{1}{\mathcal{F}_\lambda(k_1, k_2; y_1, \psi)} \\ & \times \left\{ \ln|\psi + k_2| + \ln|2y_1(\psi - k_2)(\lambda^2 - k_1^2 + k_2^2) - 4k_1^2(\psi - k_2)(\psi - y_1)| \right. \\ & \left. + (\lambda^2 + k_1^2 + k_2^2 - 2y_1 k_2) \mathcal{F}_\lambda(k_1, k_2; y_1, \psi) + \mathcal{F}_\lambda^2(k_1, k_2; y_1, \psi) \right\}. \end{aligned} \quad (B.7b)$$

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Figure Captions

Fig. 1. Goldstone diagrams for the self-energy, with the pion dressing the particle (a) or the hole (b) line.

Fig. 2. The self-energy contribution to the longitudinal response function is shown as a function of the energy transfer ω for three values of the momentum transfer: $q=300, 500, \text{ and } 1000 \text{ MeV/c}$. The dashed lines represent the separated particle and hole terms, the solid line being their sum. Here and in the following figures, unless otherwise specified, the nucleus considered is ^{12}C , $k_F=225 \text{ MeV/c}$ and $\Lambda_\pi=1.3 \text{ GeV}$.

Fig. 3. The Hartree-Fock longitudinal response (solid curve) is displayed and compared to the free RFG (dotted) and first-order self-energy correlated (dashed) responses. The first and last almost coincide.

Fig. 4. Goldstone diagrams corresponding to the forward-going (a) and backward-going (b) terms of the exchange correlations, defined in eqs. (3.3) and (3.4). In the text these are labeled “ F ” and “ B ”, respectively.

Fig. 5. The exchange contribution to the longitudinal response is shown (solid curves). The forward- (dashed) and backward-going (dot-dashed) terms, “ F ” and “ B ”, respectively, are also separately displayed.

Fig. 6. The same as fig. 5, but now for the transverse response.

Fig. 7. Goldstone diagrams entering into the Tamm-Dancoff series for the polarization propagator with pion exchange.

Fig. 8. The free (dotted), first-order exchange (dashed) and first continued fraction (solid) longitudinal responses are plotted as functions of ω at $q=500$ and 1000 MeV/c . For the results in the left-hand panel, the interaction is the pure “ δ -like” part of the pion potential, while for the results in the right-hand panel the full potential is taken.

Fig. 9. The free Fermi gas longitudinal response is shown as dotted curves; the dashed curves include the self-energy and exchange correlations, whereas the solid curves also include MEC.

Fig. 10. The same as fig. 9, but now for the transverse response.

Fig. 11. The shift of the quasielastic peak due to pionic correlations is shown as a function of q in the longitudinal (solid curve) and transverse (dashed curve) channels.

Fig. 12. The longitudinal (upper panel) and transverse (lower panel) sum rules $S_L(q)$ and $S_T(q)$, as defined through eq. (4.5), are shown as functions of q . The dotted curves correspond to the RFG, the dashed curves include self-energy and exchange correlations and the solid ones contain as well the MEC contribution. In the small windows the ratios of the correlated sum rules to the free ones are displayed: the dashed curves do not include the MEC, the solid ones do.

Fig. 13. Upper panel: the isoscalar free (dashed) and correlated (solid) longitudinal responses are displayed as functions of ω at $q=500$ MeV/c. Lower panel: the same quantities are shown in the isovector channel.

Fig. 14. The longitudinal weak neutral current response R_{AV}^L at $q=500$ MeV/c, showing the free RFG (dashed) and correlated (solid) results.

Fig. 15. The exchange contribution to the longitudinal response at $q=500$ MeV/c is displayed in the upper panel for three different values of Λ_π : 10 GeV (dashed), 1.3 GeV (dot-dashed) and 0.8 GeV (dotted). In the lower panel the longitudinal self-energy is shown, with the same meaning for the three curves.

Fig. 16. The exchange (upper panel) and MEC (lower panel) contributions to the transverse response at $q=500$ MeV/c are displayed for $\Lambda_\pi=10$ GeV (dashed), 1.3 GeV (dot-dashed) and 0.8 GeV (dotted).

Fig. 17. The self-energy contribution to the longitudinal response divided by the function $K_L(\tau, \psi_r; \eta_F)$ (eq. 5.6a) at $q=500$ MeV/c is displayed as a function

of the scaling variable ψ_r ; three different values of k_F have been chosen: 200 MeV/c (dashed), 225 MeV/c (dot-dashed) and 250 MeV/c (solid).

Fig. 18. The forward-going (F , upper panel) and backward-going (B , lower panel) terms of the exchange, divided by $K_L(\tau, \psi_r; \eta_F)$ in the longitudinal channel shown as functions of ψ_r for $k_F=200$ MeV/c (dashed), 225 MeV/c (dot-dashed) and 250 MeV/c (solid) at $q=500$ MeV/c.

Fig. 19. The forward-going (F , upper panel) and backward-going (B , lower panel) terms of the exchange, divided by $K_T(\tau, \psi_r; \eta_F)$ (eq. 5.6b) in the transverse channel are shown as functions of ψ_r for $k_F=200$ MeV/c (dashed), 225 MeV/c (dot-dashed) and 250 MeV/c (solid) at $q=500$ MeV/c.

Fig. 20. The MEC contribution to the longitudinal response divided by $K_L(\tau, \psi_r; \eta_F)$ is displayed in the upper panel as a function of ψ_r for $k_F=200$ MeV/c (dashed), 225 MeV/c (dot-dashed) and 250 MeV/c (solid) at $q=500$ MeV/c. In the lower panel the transverse MEC contribution divided by $K_T(\tau, \psi_r; \eta_F)$ is shown for the same set of k_F values.

Fig. 21. Gauge invariance in many-body theories. Diagram (a) (self-energy) has the same momentum flow in the two electromagnetic vertices. Diagrams (b) (ring), (c) (exchange), (d) (exchange contributions to antisymmetrized RPA) and (e) (ladder) have not. The first class of diagrams fulfills gauge invariance, the second does not.